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MEMORANDUM

HEAT TRANSFER FROM CYLINDERS IN TRANSITION FROM
SLIP FLOW TO FREE-MOLECULE FLOW

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HEAT TRANSFER FROM CYLINDERS IN TRANSITION FROM
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SUMMARY

Over 600 measured heat-transfer coefficients in the transition from slip to free-molecule flow have been correlated by using the Nusselt number Nu as a function of the Knudsen Kn and Reynolds Re (or Mach M) numbers. The experimental range for these heat-transfer data from transverse cylinders in air corresponds to the following dimensionless groups: M , 0.10 to 0.90; Re , 0.03 to 11.5; Kn , 0.10 to 5.0. The total air temperature T_t was maintained constant at 80° F, but wire temperature was varied from 150° to 580° F. At $Kn = 0.10$, Nu extrapolates smoothly into slip-flow empirical curves that show Nu as a function of Re and M or Kn . The correlation gradually changes from the $\sqrt{Re_t}$ dependence characteristic of continuum flow to first-power Re dependence as Kn increases (decreasing Re). At the experimental limit $Kn = 5.0$, the Nu data correlate with a mean fractional error of ± 13 percent by the prediction of free-molecule-flow theory. In comparing experimental results with theory, an accommodation coefficient of 0.57 ± 0.07 was inferred from the heat-transfer data, which were obtained with etched tungsten wire in air.

The wire recovery temperature T_e was measured and compared with existing data and theory in terms of a ratio $\eta (\equiv T_e/T_t)$. The results can be divided into three groups by Kn criteria: For $Kn < 2.0$, η is independent of Kn , and η decreases from 1.0 to 0.97 as M increases from 0 to 0.90; for $2.0 < Kn < 5.0$, η is a function of both Kn and M in this transition region to fully developed free-molecule flow; and for $Kn > 5.0$, η predicted by free-molecule-flow theory is observed and η increases from 1.0 to 1.08 as M increases from 0 to 0.90, again independent of Kn . Therefore, these T_e data provide a guide to the boundary of fully developed free-molecule flow, which is inferred from this research to exist for $Kn > 5.0$. This boundary criterion is substantiated by other published data on T_e at supersonic speeds.

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The application of fine hot wires to pressure measurement in subsonic rarefied flows is discussed. It should be possible to develop this Pirani-type gage for air pressures below 1900 microns of mercury.

INTRODUCTION

Interest in heat transfer to rarefied gas flows has grown with the advent of high-altitude satellites and missiles. The fundamental heat-transfer data presented in this report were obtained with transverse cylinders in subsonic airflows. The importance of this experimental work to problems of high-speed flight is that these heat-transfer data were obtained in the transition from slip to free-molecule flow. No theoretical approach has been successful in this transition region (ref. 1); and, as will be discussed shortly, experimental results in this flow regime are meager.

Another possible application of this research is for the development of instrumentation for low-density facilities. The conventional hot-wire anemometer, which has been a valuable tool for studying turbulent velocity fluctuations in aerodynamic research, may be valuable in low-density flow studies as a pressure gage. This application is discussed in the text.

Experimental Range of This Research

The usual dimensionless parameters used to express the gas-flow regime are the Mach number and the Reynolds number. Figure 1 is a convenient summary of recent heat-transfer experiments with transverse cylinders as the heat-transfer element. The Mach number is based on free-stream velocity and static temperatures; the Reynolds number is based on the cylinder diameter and on free-stream velocity, density, and viscosity. The shaded areas represent the work of previous investigators in the rarefied-gas regions, and these areas are keyed by numbers to the reference list of this report (refs. 2 to 16). A recent review of the previous work is given in reference 2.

Another dimensionless group, the Knudsen number Kn , is useful in rarefied-gas flows. The kinetic theory of gases relates the Knudsen number (ratio of mean free path of gas to cylinder diameter) to the ratio of Mach number to Reynolds number. For air,

$$\text{Kn} \equiv \frac{\lambda}{D_w} = 1.46 \frac{M}{\text{Re}_t} \sqrt{\frac{T}{T_t}} \quad (1)$$

(All symbols are defined in appendix A.)

The constant Knudsen number lines in figure 1 provide an approximate guide to the three broad divisions of airflow. Although these regions are probably not sharply defined, reference 14 proposes the following boundaries for flow over transverse cylinders:

- (1) Continuum flow: $\text{Kn} < 0.001$
- (2) Slip flow: $0.001 < \text{Kn} < 2.0$
- (3) Free-molecule flow: $\text{Kn} > 2.0$

More recently, recovery-temperature data from cylinders in the low-density supersonic tunnel at the University of California (refs. 16 and 17) have indicated that fully developed free-molecule flow may not be attained until a Knudsen number of about 5.

The experimental range of the research reported here is also shown in figure 1. The boundaries shown are for Knudsen numbers from 0.10 to 5.0, Reynolds numbers from 0.03 to 11.50, and Mach numbers from 0.10 to 0.90.

Objectives of This Research

The primary objective of this work is to present an empirical Nusselt number correlation of heat-transfer data obtained in the transition from slip to free-molecule flow. The need for an experimental approach in this region has already been mentioned. At the lower Reynolds number limit of this research where fully developed free-molecule flow is approached, it will be possible to compare predictions of the theory for very rarefied-gas flow with experimental observations. No verification of free-molecule-flow analysis for heat transfer has been published for subsonic and transonic flows; verification to date has been limited to supersonic streams (refs. 14 and 16). That this research will fill an existing gap in the literature can be seen readily in figure 1.

Another main objective of this research is to obtain data on the equilibrium or recovery temperature of a cylinder in the transition to free-molecule flow. No data in subsonic flows have been reported previously. These measurements serve as a direct test (independent of empirical constants) for the boundary of the free-molecule-flow regime.

Specification of the Knudsen number above which the theory of very rarefied-gas flow applies is also an objective.

Finally, an attempt will be made to infer an accommodation coefficient for tungsten wire in air from the heat-transfer data. The significance of this empirical constant is discussed in the text.

APPARATUS AND PROCEDURE

The equipment and procedure used in this research are similar to those described in reference 2. Therefore, this section will present only an outline, particularly pointing out significant departure from the methods of the previous report.

Apparatus

Tunnel and air facility. - A sketch of the variable-density, low-turbulence tunnel is shown in figure 2. Dry air at ambient temperature passed through the cone-shaped filter before entering the $1\frac{1}{2}$ -inch-diameter test section. The insert in figure 2 shows a detail of the packing gland, which provided a vacuum seal around the probe but allowed the probe to be moved out of the tunnel for airflow adjustments.

The air velocity in the test section was calculated from the mass-flow rate, which was metered by a calibrated sonic orifice upstream of the filter. Two orifices of different capacity were used in the course of the experiment.

The tunnel was serviced by the central air facilities at the Lewis Research Center. The exhaust pressure is normally 4.0 inches of mercury absolute, but this was extended to 0.05 inch absolute by two stream ejectors in series downstream of the test section. The static pressure in the test section could be varied from 0.05 to 4.0 inches of mercury. A dibutylphthalate U-tube manometer was used to measure the static pressure in the test section by referencing one arm of the U to a 50-micron reservoir. The pressure upstream of the sonic orifice was read from calibrated mercury manometers.

The total air temperature was $80^{\circ} \pm 5^{\circ}$ F, and it was measured with a calibrated thermocouple located in the inlet of the nozzle (fig. 2).

Probe design and tungsten wire. - A sketch of the probe used in this investigation is shown in figure 3. Since the length of the wire was about 2500 times its diameter, flow interference of the probe prongs was minimized. Early in this investigation, it was found that air

leaked through some of the probe bodies. Careful cementing of internal probe parts was necessary to make the probe design used herein suitable for vacuum applications.

Tungsten wire was the heat-transfer element of the probe. The wire had a nominal diameter of 0.00020 ± 0.00002 inch. A diameter as determined from electron photomicrographs in reference 2 was 2.2×10^{-4} inch. However, the diameter from the spool section used in this work was calculated from electrical resistance measurements to be intermediate between the nominal diameter and that used in reference 2. A diameter of 2.11×10^{-4} inch was used for the calculations in this report.

One of the most important physical properties of tungsten in the calculations of heat-loss data is the relation between electrical resistance and temperature. Seven wire samples were silver-soldered to probes, annealed, and then placed in a small calibration heater (fig. 4). At various air temperatures in the heater, the wire resistance was determined by balancing a Kelvin double bridge while passing a small detection current (which did not heat the wire within the sensitivity of measurement) through the wire. A sample calibration curve is given in figure 5. A least-square solution for the best parabola

$\left\{ R = R_0 \left[1 + \alpha_w (T - 32) + \beta_w (T - 32)^2 \right] \right\}$ through the calibration points yielded the coefficients shown in figure 5. In this manner, the average second-order coefficient β_w was found to be $2.67 \times 10^{-7} \text{ } ^\circ\text{F}^{-2}$. For all wires subsequently used in the tunnel, only a partial calibration from 32° to 200° F was determined. Then assuming a value of $\beta_w = 2.67 \times 10^{-7} \text{ } ^\circ\text{F}^{-2}$, a least-square solution for R_0 and α_w was calculated for each wire used. Table I summarizes the physical constants of the wire samples used in the heat-loss measurements.

Anemometer electrical equipment. - The fine-wire probe formed one arm of a Kelvin double bridge. The bridge described in reference 9 was used; the only modification was to change the standard resistor to 100 ohms. The electrical power into the bridge for hot-wire operation was controlled by the constant-average-temperature servo described in reference 2. The desired hot-wire resistance was set on the bridge, and the power necessary to heat the wire to this resistance was calculated from the voltage drop measured across the hot wire by a potentiometer.

The recovery resistance was measured by balancing the bridge when a negligible detection current was flowing through the wire. The use of the Kelvin double bridge made it possible to determine the wire resistance while almost completely eliminating lead resistances from the measurements.

Procedure

The tunnel was operated at various Mach numbers from 0.10 to 0.90. The range of Knudsen number based on wire diameter was 0.10 to 5.0, which corresponds to static air densities from about 0.0090 to 0.0002 pound (mass) per cubic foot. An important feature of the procedure was a rather random schedule for taking data, as can be seen in table II from the chronological run numbers. This reduced the probability of systematic error in the measurements.

After tunnel conditions had been adjusted, the probe was moved into the test section, and manometer readings and temperature were recorded. The recovery resistance was measured, after which six progressively higher hot-wire resistances were set on the bridge, and the power input to the wire was determined for each setting. Then the recovery temperature was again recorded along with tunnel operating conditions.

DIMENSIONLESS GROUPS OF CORRELATION

This section is devoted to a brief explanation of the dimensionless parameters used in correlating the data.

The convective heat-loss rate defines the heat-transfer coefficient h :

$$Q_C = h\pi D_W l (T_W - T_e) \quad (2)$$

The heat-transfer coefficient is defined in terms of the recovery temperature T_r ; thus, for the adiabatic wire, $T_r = T_e$.

The Nusselt number is defined as

$$Nu_t = \frac{hD_W}{k_t} = \psi \frac{J' V_W^2}{\pi l k_t \Omega_W (T_W - T_e)} \quad (3)$$

The air thermal conductivity k_t is evaluated at total air temperature using the values given in reference 18. Two published correction procedures were used to account for the heat loss by conduction from the hot wire to the supports. Each procedure gave a correction factor ψ ; and, as discussed in reference 2, this factor was nearly identical in numerical value for both end-loss computations. Radiant heat loss from the wire was calculated to be negligible for all operating conditions.

The wire temperature for both hot-wire operation and recovery-temperature measurements was calculated from the resistance-temperature calibration:

$$\Omega = \Omega_0 \left[1 + \alpha_w(T - 32) + \beta_w(T - 32)^2 \right] \quad (4)$$

The Reynolds number for the wire was defined by the wire diameter and the free-stream (or static) density and velocity. The air viscosity μ_t was evaluated at total air temperature using values given in reference 18:

$$Re_t = \frac{\rho U D_w}{\mu_t} \quad (5)$$

The Knudsen number was calculated from free-stream density and the wire diameter using the formula suggested by reference 13:

$$Kn = \frac{\lambda}{D_w} = \frac{1.5870 \cdot 10^{-8}}{\rho D_w} \quad (6)$$

The constant 1.5870×10^{-8} has the units (lb mass)/(sq ft). Equation (6) assumes that the mean free path for air λ is given by elementary kinetic theory and that $\rho \lambda$ is a constant.

The Mach number was calculated from the velocity measured with a sonic orifice and the static temperature:

$$M = \frac{U}{49.02 \sqrt{T}} \quad (7)$$

The data reduction of this research was performed on the IBM 653 digital computer.

RESULTS AND DISCUSSION

The presentation and discussion of results is divided into three parts. In the first section, the recovery-temperature results are presented and are compared with other available recovery temperature data and theoretical predictions of recovery temperature. In the second section, the heat-transfer results are given under the heading "Nusselt Number Correlation." Finally, the application of this research to low-density flow instrumentation is examined.

Recovery Temperature

A survey of published data on recovery temperature of transverse cylinders in subsonic airflows is summarized in figure 6. Each curve is keyed to the reference source. These data were obtained over a wide Reynolds number range. Reference 12 data extended to $Re_t = 30,000$ or $Kn = 4 \times 10^{-5}$, which is well into continuum flow. The data from reference 13 were obtained in slip flows to a limiting Knudsen number of 0.10. Previous investigations (e.g., refs. 12 and 13) indicate that the recovery temperature is a function only of Mach number and is nearly independent of Knudsen number within experimental scatter for at least $4 \times 10^{-5} < Kn < 0.10$.

However, both theory (ref. 14) and previous experiments (refs. 14, 16, 17, and 19) have shown that the temperature of a transverse cylinder will exceed the total temperature of the gas stream in which it is located when the cylinder is placed with its axis perpendicular to a uniform gas stream in which the mean free path is several times greater than the cylinder diameter. This temperature is a function only of the Mach number, static air temperature, and the number of excited degrees of freedom of the gas molecules. The equation for the recovery temperature of an infinite cylinder $T_{e,\infty}$ placed in a diatomic gas flow is given in references 14 and 19 as

$$\frac{T_{e,\infty}}{T_t} = \frac{T}{T_t} \frac{f(s)}{g(s)} \quad (8)$$

where $f(s)$ and $g(s)$ are dimensionless functions of the molecular speed ratios s . The molecular speed ratio is related to the more familiar Mach number as follows:

$$s = \sqrt{\frac{\gamma}{2}} M \quad (9)$$

For convenience, the functions $f(s)$ and $g(s)$ are tabulated over the range of interest in this report in table III.

Equation (8) is plotted in figure 7. It is important to notice that for free-molecule flow the theory predicts that the recovery-temperature ratio η is a function only of Mach number. However, as was pointed out in the INTRODUCTION, the boundary for fully developed free-molecule flow is uncertain.

The recovery-temperature results of this research were obtained with a finite length of wire. An equation (B9) is derived in appendix B to account for conduction heat loss from a wire to its supports. The maximum departure from infinite wire behavior for the 0.50-inch tungsten wires used for this research was calculated to be 2 percent. Therefore, no correction has been applied to the measured recovery-temperature

data, which will be assumed to be obtained from infinite wires. This statement does not apply to hot-wire operation in the determination of Nusselt number.

The results obtained in this research are given in table IV and are shown in figure 8, in which the ratio of recovery temperature to total temperature is compared with the curves from continuum- and slip-flow experiments (fig. 6) and free-molecule-flow theory (fig. 7). The data on the recovery-temperature ratio η can be divided into three regions: The first is for $Kn < 2.0$, where η is only Mach number dependent and decreases from 1.0 to 0.97 as Mach number increases from 0 to 0.90; a transition region is then discernible in which η is a function of both Mach and Knudsen numbers, and the constant Kn lines begin to approach the predicted theoretical curve for fully developed free-molecule flow; finally, for Kn greater than about 5.0, η is again independent of Kn and increases from 1.0 to 1.08 as Mach number increases from 0 to 0.90.

Figure 9 shows a comparison of the measured recovery temperature T_e of figure 8 and the theoretical recovery temperature $T_{e,\infty}$ in free-molecule flow. The ratio of T_e to $T_{e,\infty}$ is shown here as a function of Knudsen number. The value of Knudsen number where $T_e/T_{e,\infty}$ is equal to unity is a criterion for the attainment of fully developed free-molecule flow. This value appears to be about $Kn = 5.0$ in figure 9. It is important to recall that this result is independent of empirical constants (i.e., accommodation coefficient), as can be seen in equation (8). Approximately the same value of Knudsen number ($Kn = 5.0$) for fully developed free-molecule flow was obtained in the work of references 16 and 17, although these studies were carried out for supersonic flows. The inference that the boundary for free-molecule flow may be taken as $Kn > 5.0$ appears to be contrary to the conclusions from the pioneer work of reference 19. Reference 19 proposed that the boundary criterion was $Kn > 2.0$. A close look at the data of reference 19 shows that lack of sufficient data in the range $1.0 < Kn < 5.0$ could easily account for the difference in the proposed criterion. Finally, it should be pointed out that the dashed lines of constant Mach number for $0.10 < Kn < 1.0$ in figure 9 connect the present data with values calculated from the reference 9 curve in figure 6.

Nusselt Number Correlation

Before presenting the heat-transfer results of this investigation, it will be convenient to outline the form that the Nusselt number correlation should be expected to take in the transition from slip to free-molecule flow. Figure 10 (from ref. 2) is an attempted Nusselt number correlation in subsonic continuum, slip, and free-molecule flow. In

figure 10, the Nusselt number is shown as a function of Reynolds number; constant Mach number and Knudsen number parametric curves are shown solid and dashed, respectively.

For Reynolds numbers between 250 and 30,000, the empirical correlation given in reference 12 is plotted. The important feature of this relation for cylinders in continuum flow is that the Nusselt number is well correlated by $\sqrt{Re_t}$. Note that changing the subsonic Mach number has small percentage effect on Nu_t at constant Re_t .

The correlation shown for $1.0 < Re_t < 100$ in figure 10 is based on the slip-flow data from reference 2. The increasing necessity for an additional parameter other than the Reynolds number to correlate the experimental heat-transfer data shows clearly in this slip-flow region as Re_t decreases. This additional parameter is either the Mach number or the Knudsen number. Both forms that this correlation might take are shown in figure 10; that is, $Nu_t = f(Re_t, M)$ and $Nu_t = f(Re_t, Kn)$ are both given. However, only one additional parameter (M or Kn) is independent, since these groups are related by equation (1). Since the need for the additional parameter is the result of the slip or rarefied-gas phenomenon, it seems preferable to view the Knudsen number as the additional independent parameter.

The Nusselt number correlation shown in figure 10 for $Re < 1.0$ is a prediction of free-molecule-flow theory (using an accommodation coefficient $\alpha = 0.90$), which will be discussed in detail later. Note that the theory predicts first-power Reynolds number dependence and a large separation of the constant M parametric curves. Furthermore, it is clear that, although the slip-flow data extrapolate smoothly into the continuum curves in the region $Re_t \approx 100$, the free-molecule-flow prediction does not agree quantitatively with the slip-flow data around $Re_t \approx 1.0$. With this discussion as a background, the results of this research will now be examined.

A representative portion of the heat-transfer data obtained in this experiment is shown in figure 11. These data were obtained at a total air temperature of 80°F and a cylinder temperature of 580°F . Both forms of the Nusselt number correlation are shown in figure 11; that is, $Nu_t = f(Re_t, Kn)$ and $Nu_t = f(Re_t, M)$. It will be convenient to discuss figure 11 from two points of view: first, to compare the correlation at $Kn = 0.109$ with the slip-flow results presented in figure 10; then, to compare the data at $Kn = 4.70$ with free-molecule-flow theory. The general lack of scatter in the data, as shown in figure 11 for $0.109 < Kn < 4.70$, makes it unnecessary to discuss the internal consistency of the data between the experimental limits.

Figure 12 compares the lower Knudsen number portion of the data from this report with the slip-flow data from reference 2. The two sets of data agree quite well, and the same trends noted in reference 2 are present here. These are (1) a gradual change in slope of the constant Mach number parametric lines from fractional-power to first-power Reynolds number dependence as Re decreases; (2) a large separation of the constant Mach number curves, showing decreasing dependence of heat transfer on air velocity, especially in low subsonic flows; and (3) the increasing importance of the Knudsen number as the governing parameter for subsonic flows as Re decreases.

Before comparing the high Knudsen number data with free-molecule-flow predictions, it is convenient to review the results of the free-molecule-flow analysis for heat transfer from cylinders, which is given in detail by references 14 and 19. Reference 19 gives the following equation for the heat-transfer coefficient h :

$$h = \frac{pv_m \alpha [g(s)]}{2J(\pi^{3/2})T} \quad (10)$$

Reexpressing equation (10) in the more conventional terms of Nusselt number shows that

$$Nu_t = \left[\left(\frac{R\mu_t}{2J\pi k_t} \right) \alpha \right] \frac{g(s)}{Kn} \sqrt{\frac{T}{T_t}} \quad (11)$$

The accommodation coefficient α is a way of expressing the efficiency of the energy-exchange process that occurs between a solid surface and an impinging molecular stream (ref. 20). The definition can be written as follows:

$$\alpha = \frac{E_i - E_r}{E_i - E_w} \quad (12)$$

At present, the accommodation coefficient can be considered an empirical constant that depends not only on the metal and its surface finish but also on the gas. Generalizations concerning the accommodation coefficient are scant and not without exception. Therefore, α can be regarded as an undetermined constant when equation (11) is evaluated for a total air temperature of 80° F.

The following equation predicts Nu_t as a function of the Knudsen and Mach numbers where the free-molecule-flow theory applies (i.e., $Kn > 5.0$) and the accommodation coefficient is known:

$$Nu_t = 0.03214(\alpha) \frac{g(s)}{Kn} \sqrt{\frac{T}{T_t}} \quad (13)$$

In order to estimate the accommodation coefficient of the tungsten wire in air, it was necessary to assume that equation (13) is valid for $Kn = 4.00$ and 4.70 , which are the limits of the experimental data. Table V summarizes the results of solving equation (13) for α using the Nusselt number measurements of this experiment. An accommodation coefficient of 0.57 ± 0.07 for tungsten wire in air was determined in this manner.

It is now possible to compare the high Knudsen number portion of the heat-transfer data with free-molecule-flow predictions. Figure 13 shows this comparison. In evaluating the theoretical curves from equation (13), an accommodation coefficient of 0.57 has been used for all flow conditions. From figure 13, the agreement between theory and experiment can be seen to be adequate for most engineering purposes. More specifically, equation (13) and $\alpha = 0.57$ predicted the observed Nusselt numbers at $Kn = 4.00$ and 4.70 with an average deviation of a single observation of ± 0.00537 , which corresponds to a mean percentage error in Nusselt number of about ± 13 percent.

Equation (13) suggests another method of presenting heat-transfer data in free-molecule flow. Figure 14 is a plot of equation (13) with $\alpha = 0.57$. This figure will be referred to in the next section.

Only a portion of the data given in table II are shown in figure 11. The remainder of the data were obtained at wire temperatures lower than 580°F but at the same flow conditions. This discussion of the results at lower wire temperatures has been deferred until now because these data generally only verify the results at $T_w = 580^\circ \text{F}$. A series of plots substantiating this statement is given in figure 15. The general appearance of figure 15 at $T_w = 580^\circ \text{F}$ is repeated in all six parts at the lower wire temperatures. The data scatter increases with decreasing wire temperature; but it is clear that, if the wire temperature affects the measured Nusselt numbers, it is a secondary effect.

Application to Low-Density Instrumentation

Fine wires have been used as hot-wire anemometers to measure time-mean and turbulent fluctuating air velocities for many years. During the majority of this time, it was not recognized that the flow over the sensitive element was a rarefied-gas flow. That is, even though the air was at atmospheric density, the fine wire was in slip flow. Thus, hot-wire calibrations based on King's equation (e.g., see ref. 21) are somewhat in error, and this fact has been discussed in recent reports (refs. 2 and 13). On the other hand, fine hot wires are the sensitive element of pressure-measuring gages that were developed before the advent of the anemometer in 1914. The Pirani or thermal-conductivity gages are hot

wires whose heat loss is calibrated in pressure units, usually in the range 10 to 200 microns of mercury. Since both anemometer and Pirani gage applications depend on the heat-loss characteristics of fine wires, it is appropriate to relate the data of this research to these instruments.

The convective heat loss from cylinders over a wide range of air-flow conditions may be predicted by using figure 16. For flow of air at ambient temperature and pressure, the Knudsen number for a 0.00020-inch-diameter wire is about 0.012. Therefore, the measured convective heat-loss rate (i.e., Nu_t) may be used in principle to infer an air velocity (from either M or Re) by using figure 16. In practice, an instrument calibration, which is obtained within the restrictive limits of application, is substituted for figure 16. However, it is not anemometer applications that are to be discussed here. The purpose of this discussion is to point out the gradual change in heat-loss characteristics that makes the fine hot wire applicable to pressure measurements as the flow over the wire changes from slip to free-molecule flow. This transition to very rarefied airflow causes the change in heat-transfer-coefficient calibration shown in figure 16 as Knudsen number increases from about 0.012 to 5.0. The velocity sensitivity of the wire decreases monatomically as the Knudsen number increases; and, when the flow over the wire becomes a fully developed free-molecule flow, the velocity sensitivity has all but vanished in the subsonic regime. This point is best observed in figure 14. Note that, at any constant air density (i.e., constant $1/Kn$, fig. 14), the percentage change in heat-transfer rate, as given by Nu_t , is very small due to changes in airflow rate from stagnant air ($M = 0$) to about $M = 0.4$. More specifically, using equation (13) and the tabulated values of $g(s)$ in table III, the heat-transfer rate is calculated to increase only about 5 percent as M varies from 0 to 0.40. Recalling that equation (13) can be applied for all $Kn > 5.0$ according to this research, then it is apparent that a 0.00020-inch-diameter hot wire could be used as an air pressure gage for flowing air at room temperature below about 1900 microns of mercury. This instrument would require less than a 5-percent air velocity correction for $U < 400$ feet per second; the corrections for heat conduction to wire supports and radiation to surroundings are usually greater than 5 percent for Pirani gages (e.g., see ref. 22).

In the previous paragraph, a possible extension of Pirani-type gages to pressure measurements with the sensitive element placed in subsonic airflows has been suggested. Naturally, the principle would apply equally well to any gas in a free-molecule flow if the appropriate monatomic or diatomic gas equation is used (ref. 19). Finally, the application of equation (13) to instrumentation in supersonic free-molecule flows has been the subject of several University of California (Berkeley) reports (refs. 16, 17, and 23). It is interesting to note that, for Mach numbers greater than about 2, the hot-wire sensitivity

is governed by the first power of the Reynolds number almost independent of Mach number. Thus, for $M > 2.0$, the fine wire is almost equally sensitive to air density and to velocity in a free-molecule flow.

CONCLUDING REMARKS

The following conclusions can be drawn from the heat-transfer measurements reported herein in the transition region from slip to free-molecule flow:

1. The Nusselt number Nu_t correlation for transverse cylinders in subsonic airstreams is complicated in the transition regime from slip flow to free-molecule flow. No simple-power-law engineering equation has been found for this Nusselt number correlation, but it is conveniently given in graphical form (fig. 11). Figure 11 shows Nu_t as a function of Reynolds number Re_t with constant Mach number M and Knudsen number Kn parametric lines. The three important features of this correlation are:

- (a) The gradual change in slope of the constant M parametric lines from fractional-power to first-power Reynolds number dependence as Re_t decreases
- (b) Large separation of the constant M curves showing decreasing dependence of heat transfer on air velocity
- (c) The increasing importance of Kn as the governing parameter for subsonic flows as Re_t decreases.

2. The measured Nusselt numbers of this report are compared with published slip-flow data at $Kn = 0.077$ and free-molecule theory at $Kn > 5.0$ in figures 12 and 13, respectively. The agreement at low Kn with slip-flow data is good; and, by assuming an accommodation coefficient α of 0.57, the data agree adequately with predictions of free-molecule-flow theory. That is, for $Kn > 4.0$, the data are correlated with an average deviation of ± 13 percent by

$$Nu_t = 0.03214 (\alpha) \frac{g(s)}{Kn} \sqrt{\frac{T}{T_t}}$$

where $g(s)$ is a Mach number function defined by theory, T is static temperature, and T_t is total air temperature.

3. An average accommodation coefficient α of 0.57 ± 0.07 can be inferred from the measured heat-transfer coefficients of etched tungsten wire in subsonic airflows.

4. The transition to fully developed free-molecule flow appears to be complete at $Kn = 5.0$ for transverse cylinders in subsonic flow. This conclusion is based on the comparison of measured recovery temperature with theoretically predicted recovery temperature as shown in figure 9. The inference does not depend on empirical constants; that is, it is independent of accommodation coefficient.

5. The recovery-temperature data of this report can be divided into three regimes by Knudsen number criteria, as can be seen in figure 8: $Kn < 2.0$, the ratio of recovery to total air temperature $\eta \equiv (T_e/T_t)$ is independent of Kn and η decreases from 1.0 to 0.97 as Mach number increases from 0 to 0.90; $2.0 < Kn < 5.0$, η is a function of both Kn and M in this transition region to fully developed free-molecule flow; $Kn > 5.0$, η predicted by free-molecule flow theory is observed and η increases from 1.0 to 1.08 as M increases from 0 to 0.90, again independent of Kn .

Lewis Research Center

National Aeronautics and Space Administration

Cleveland, Ohio, January 30, 1959

APPENDIX A

SYMBOLS

| | |
|------------------------|---|
| A_w | cross-sectional area of cylinder, $\pi D_w^2/4$, sq ft |
| Bi | Biot number, hD_w/K_w , dimensionless |
| D_w | wire diameter, ft or in. |
| E_i | rate of incident molecular energy |
| E_r | rate of reemitted molecular energy |
| E_w | rate of reemitted molecular energy that would be carried by the scattered stream if it were a stream issuing from a gas in equilibrium at the surface temperature T_w |
| $f(s)$ | dimensionless function of s ; $f(s) = \pi \left[\exp \left(-\frac{s^2}{2} \right) \right] \times \left\{ (s^2 + 3) I_0 \left(\frac{s^2}{2} \right) + \left(s^2 + \frac{7}{2} \right) s^2 \left[I_0 \left(\frac{s^2}{2} \right) + I_1 \left(\frac{s^2}{2} \right) \right] \right\}$ |
| $g(s)$ | dimensionless function of s ; $g(s) = 3\pi \left[\exp \left(-\frac{s^2}{2} \right) \right] \times \left[(1 + s^2) I_0 \left(\frac{s^2}{2} \right) + s^2 I_1 \left(\frac{s^2}{2} \right) \right]$ |
| g_c | unit conversion factor, $(\text{lb(M)})(\text{ft})/(\text{lb(F)})(\text{sec}^2)$ |
| h | convective heat-transfer coefficient, $\text{Btu}/(\text{sec})(\text{sq ft})(^\circ\text{F})$ |
| $I_0(z)$, $I_1(z)$ | modified Bessel function of first kind, zero and first order, respectively, dimensionless |
| J | conversion factor, $\text{ft-lb(F)}/\text{Btu}$ |
| J' | conversion of watts to Btu/sec , 9.484×10^{-4} |
| K_w | thermal conductivity of cylinder metal, $\text{Btu}/(\text{sec})(\text{ft})(^\circ\text{F})$ |
| Kn | Knudsen number, λ/D_w |
| k_t | thermal conductivity at total air temperature, $\text{Btu}/(\text{sec})(\text{ft})(^\circ\text{F})$ |
| l | length of wire, ft |

| | |
|-----------------|--|
| M | Mach number |
| m | mass of 1 molecule, $\text{lb(M)}/\text{molecule}$ |
| Nu_t | Nusselt number, hD_w/k_t |
| Nu_t'' | Nusselt number uncorrected for heat loss to supports |
| n, n' | number of molecules striking body, $\text{molecules}/(\text{sq ft})(\text{sec})$ |
| p | static pressure, $\text{lb(F)}/\text{sq ft}$ |
| Q | length-average heat-loss rate, Btu/sec |
| R | gas constant for air, $\text{ft-lb(F)}/(\text{lb(M)})(^\circ\text{F})$ |
| Re_t | Reynolds number, $\rho D_w U/\mu_t$ |
| r | wire radius, ft |
| s | molecular speed ratio, $\sqrt{\gamma/2} M$ |
| T | static or free-stream air temperature, $^\circ\text{R}$ |
| T_e | recovery or equilibrium wire temperature, $^\circ\text{R}$ |
| $T_{e,\infty}$ | theoretical recovery temperature of infinitely long cylinder in free-molecule flow, $^\circ\text{R}$ |
| T_e^* | theoretical recovery temperature of finite-length cylinder in free-molecule flow, $^\circ\text{R}$ |
| T_t | total air temperature, $^\circ\text{R}$ |
| T_w | length-average wire temperature, $^\circ\text{F}$ |
| t_w | local wire temperature at any point x, $^\circ\text{F}$ |
| U | free-stream air velocity, ft/sec |
| V_w | wire potential, volts |
| v_m | most probable molecular speed, $\sqrt{2g_cRT}$, ft/sec |
| x | any position along length of wire, $x = 0$ at wire center, ft |
| α | accommodation coefficient (eq. (12)), dimensionless |

| | |
|------------|---|
| α_w | first-order coefficient of electrical resistance - temperature relation, $^{\circ}\text{F}^{-1}$ |
| β_w | second-order coefficient of electrical resistance - temperature relation, $^{\circ}\text{F}^{-2}$ |
| γ | ratio of specific heats, 1.4 for air |
| η | recovery-temperature ratio, $\eta \equiv T_e/T_t$ |
| k | Boltzmann constant, 5.66×10^{-24} ft-lb(F)/($^{\circ}\text{F}$)(molecule) |
| λ | mean free path of air, ft |
| μ | air viscosity, lb(M)/(ft)(sec) |
| ξ | temperature variable defined by equation (B5) |
| ρ | free-stream air density, lb(M)/cu ft |
| σ | coefficient defined in equation (B3), dimensionless |
| σ_1 | coefficient defined in equation (B3), $^{\circ}\text{R}$ |
| ψ | end-loss correction ratio, dimensionless |
| Ω_w | length-average wire resistance at hot-wire temperature T_w , ohms |
| Ω_0 | length-average wire resistance at 32°F , ohms |

Subscripts:

| | |
|------|-----------------------------|
| av | average |
| C | convection |
| calc | calculated |
| meas | measured |
| t | total air temperature T_t |

APPENDIX B

ANALYSIS FOR RECOVERY TEMPERATURE OF FINITE TRANSVERSE

CYLINDER IN FREE-MOLECULE FLOW FIELD

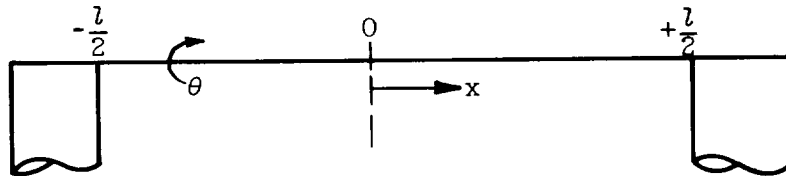
In order to compare theory with experiment accurately, it is necessary to recognize that the experimental cylinder has finite length. When the cylinder is operated as a hot wire by electrical heating, it is convenient to make a correction to the measured power input to account for the heat-loss rate to the supports. Such a procedure has been used in this report and in most previous hot-wire heat-loss reports to eliminate the aspect ratio (l/D_w) of the wire as a parameter (e.g., see ref 2). However, a different approach is useful when considering the recovery temperature of a cylinder in a free-molecule flow field. It is more convenient to account for the conduction loss to the supports in a theoretical analysis and thereby obtain the recovery temperature predicted for a given finite cylinder length. Then, the measured recovery temperature of the wire can be compared with the theoretical predictions for the particular finite cylinder. The purpose of this appendix is to develop an expression for the theoretical recovery temperature of a finite cylinder in a free-molecule flow field.

The calculation of the heat-transfer process between the cylinder and the diatomic gas has been given in detail in reference 19. Neglecting the radiant heat loss and any electrical power input, the energy balance equation on the entire finite cylinder is as follows:

$$2 \int_0^{l/2} \alpha D_w \left\{ \int_0^{\pi/2} n \left[\frac{mU^2}{2g_c} + (\psi + 1)\kappa T \right] d\theta + \int_0^{\pi/2} n' \left[\frac{mU^2}{2g_c} + (\psi' + 1)\kappa T \right] d\theta \right\} dx -$$

$$2 \int_0^{l/2} 3\alpha D_w \kappa \left(\int_0^{\pi/2} n d\theta + \int_0^{\pi/2} n' d\theta \right) t_w dx + 2JK_w A_w \left. \frac{dt_w}{dx} \right|_{l/2} = 0 \quad (B1)$$

Equation (B1) corresponds with equation (13) of reference 19, but equation (B1) includes the conduction of heat to the cylinder supports. The coordinate system is



By differentiating equation (B1) with respect to x , the desired energy-balance equation is obtained as a second-order differential equation with constant coefficients:

$$\begin{aligned} \frac{d^2 t_w}{dx^2} - \frac{6\alpha r \kappa}{JK_w A_w} \left(\int_0^{\pi/2} n \, d\theta + \int_0^{\pi/2} n' \, d\theta \right) t_w \\ = - \frac{2\alpha r}{JK_w A_w} \left\{ \int_0^{\pi/2} n \left[\frac{mU^2}{2g_c} + (\psi + 1)\kappa T \right] d\theta + \int_0^{\pi/2} n' \left[\frac{mU^2}{2g_c} + (\psi' + 1)\kappa T \right] d\theta \right\} \quad (B2) \end{aligned}$$

It is convenient to introduce two symbols for the coefficients, as follows:

$$\begin{aligned} \sigma &\equiv \frac{6\alpha r \kappa}{JK_w A_w} \left(\int_0^{\pi/2} n \, d\theta + \int_0^{\pi/2} n' \, d\theta \right) \\ \text{and} \\ \sigma_1 &\equiv \frac{2\alpha r}{JK_w A_w} \left\{ \int_0^{\pi/2} n \left[\frac{mU^2}{2g_c} + (\psi + 1)\kappa T \right] d\theta + \int_0^{\pi/2} n' \left[\frac{mU^2}{2g_c} + (\psi' + 1)\kappa T \right] d\theta \right\} \end{aligned} \quad (B3)$$

All the integrals in equations (B3) are evaluated in reference 19. Using these results, it is a straightforward but lengthy procedure to show that the ratio (σ_1/σ) is the solution for the recovery temperature of an infinitely long cylinder (ref. 14):

$$\frac{\sigma_1}{\sigma} \equiv T_{e,\infty} = T \frac{f(s)}{g(s)} \quad (B4)$$

Equation (B2) can be reduced to a homogeneous equation by the following change of variable:

$$\xi \equiv \frac{\sigma_1}{\sigma} - t_w \quad (B5)$$

With the use of equation (B5), equation (B2) becomes

$$\frac{d^2 \xi}{dx^2} + \sigma \xi = 0 \quad (B6)$$

The following boundary conditions will be used to find the temperature distribution along the wire length:

(1) Since the wire is symmetrical, at the center of the wire ($x = 0$) the temperature gradient must be zero:

$$\left. \frac{dt_w}{dx} \right|_{x=0} = \left. \frac{d\xi}{dx} \right|_{x=0} = 0$$

(2) The temperature at each support ($x = \pm l/2$) is $T_{e,s}$, where $T_{e,s}$ is the recovery temperature for the wire supporting prongs. Since the supports are large compared with the fine wire, the support recovery temperature is governed by the empirical recovery-temperature ratio η for transverse cylinders in continuum and slip flow. That is,

$$T_{e,s} = \eta T_t = f(M)$$

The functional dependence on Mach number is a topic of this report (e.g., see fig. 6). In summary,

$$t_w|_{x=\pm(l/2)} = T_{e,s} = f(M)$$

The solution of equation (B6) satisfying the boundary conditions can be written in terms of t_w as

$$t_w = T_{e,\infty} - (T_{e,\infty} - T_{e,s}) \frac{\cosh \frac{\sqrt{\sigma} x}{2}}{\cosh \frac{\sqrt{\sigma} l}{2}} \quad (B7)$$

The desired length-average recovery temperature of the finite cylinder in a free-molecule flow field is obtained by integrating over the length:

$$T_e^* = \frac{1}{l} \left[\int_{-l/2}^{l/2} T_{e,\infty} dx - \int_{-l/2}^{l/2} \frac{(T_{e,\infty} - T_{e,s})}{\cosh \frac{\sqrt{\sigma} l}{2}} \cosh \sqrt{\sigma} x dx \right] \quad (B8)$$

In final form,

$$T_e^* = T_{e,\infty} - \frac{T_{e,\infty} - T_{e,s}}{\frac{\sqrt{\sigma} l}{2}} \tanh \frac{\sqrt{\sigma} l}{2} \quad (B9)$$

Equation (B9) gives the theoretical recovery temperature T_e^* of a finite cylinder in a free-molecule flow field.

Finally, it is interesting to note the significance of the term $\sqrt{\sigma} l/2$ in the conduction correction term. It can be shown that this term is related to the predicted Nusselt number for an infinite cylinder in a free-molecule flow as given by reference 14. That is,

$$\frac{\sqrt{\sigma} l}{2} \equiv \sqrt{\frac{k_t \text{Nu}_t}{K_w}} \frac{l}{D_w} \quad (\text{B10})$$

Here, $k_t \text{Nu}_t$ is the predicted heat-transfer coefficient h_{D_w} given in the text as equation (13). The square-root term on the right side of equation (B10) occurs frequently in heat-transfer calculations; and, because it is dimensionless, this term has been called the Biot number Bi (ref. 24, appendix A),

$$\frac{\sqrt{\sigma} l}{2} \equiv \sqrt{\text{Bi}} \left(\frac{l}{D_w} \right) \quad (\text{B11})$$

REFERENCES

1. Tsien, Hsue-Shen: Superaerodynamics, Mechanics of Rarefied Gases. Jour. Aero. Sci., vol. 13, no. 12, Dec. 1946, pp. 653-664.
2. Baldwin, Lionel V.: Slip-Flow Heat Transfer from Cylinders in Subsonic Airstreams. NACA TN 4369, 1958.
3. Carbon, M. W., Kutsch, H. J., and Hawkins, G. A.: The Response of Thermocouples to Rapid Gas-Temperature Changes. Trans. ASME, vol. 72, no. 5, July 1950, pp. 655-657.
4. Flock, Ernest Franklin: Ninth Monthly Report of Progress on the Development of Thermocouple Pyrometers for Gas Turbines. NBS, Sept. 6, 1946.
5. Glawe, George E., and Johnson, Robert C.: Experimental Study of Heat Transfer to Small Cylinders in a Subsonic, High-Temperature Gas Stream. NACA TN 3934, 1957.
6. Kovásznay, Leslie S. G., and Törmarck, Sven I. A.: Heat Loss of Hot-Wires in Supersonic Flow. Bumblebee Rep. No. 127, Dept. Aero., The Johns Hopkins Univ., Apr. 1950. (Contract NOrd 8036 with Bur. Ord.)

7. Laufer, J., and McCellan, R.: Measurement of Heat Transfer from Fine Wires in Supersonic Flows. Jour. Fluid Mech., vol. 1, pt. 3, Sept. 1956, pp. 276-289.
8. Laurence, James C., and Landes, L. Gene: Auxiliary Equipment and Techniques for Adapting the Constant-Temperature Hot-Wire Anemometer to Specific Problems in Air-Flow Measurements. NACA TN 2843, 1952.
9. Lowell, Herman H.: Design and Applications of Hot-Wire Anemometers for Steady-State Measurements at Transonic and Supersonic Airspeeds. NACA TN 2117, 1950.
10. McAdams, William H.: Heat Transmission. Third Ed., ch. 10, McGraw-Hill Book Co., Inc., 1954.
11. Sandborn, Virgil A., and Laurence, James C.: Heat Loss from Yawed Hot Wires at Subsonic Mach Numbers. NACA TN 3563, 1955.
12. Scadron, Marvin D., and Warshawsky, Isidore: Experimental Determination of Time Constants and Nusselt Numbers for Bare-Wire Thermocouples in High-Velocity Air Streams and Analytical Approximation of Conduction and Radiation Errors. NACA TN 2599, 1952.
13. Spangenberg, W. G.: Heat-Loss Characteristics of Hot-Wire Anemometers at Various Densities in Transonic and Supersonic Flow. NACA TN 3381, 1955.
14. Stalder, Jackson R., Goodwin, Glen, and Creager, Marcus O.: Heat Transfer to Bodies in a High-Speed Rarefied-Gas Stream. NACA Rep. 1093, 1952. (Supersedes NACA TN 2438.)
15. Winovich, Warren, and Stine, Howard A.: Measurements of the Non-linear Variation with Temperature of Heat-Transfer Rate from Hot Wires in Transonic and Supersonic Flow. NACA TN 3965, 1957.
16. Wong, H.: Design and Development of a Free-Molecule Heat Transfer Probe. HE-150-143, Inst. Eng. Res., Univ. Calif., Oct. 15, 1956. (Contract AF-33(616)-2878.)
17. Sherman, F. S.: A Low-Density Wind-Tunnel Study of Shock-Wave Structure and Relaxation Phenomena in Gases. NACA TN 3298, 1955.
18. Hilsenrath, Joseph, et al.: Tables of Thermal Properties of Gases. Cir. 564, NBS, Nov. 1, 1955.
19. Stalder, Jackson R., Goodwin, Glen, and Creager, Marcus O.: A Comparison of Theory and Experiment for High-Speed Free-Molecule Flow. NACA Rep. 1032, 1951. (Supersedes NACA TN 2244.)

20. Kennard, Earle H.: Kinetic Theory of Gases. McGraw-Hill Book Co., Inc., 1938, pp. 312-315.
21. Corrsin, Stanley: Extended Applications of the Hot-Wire Anemometer. NACA TN 1864, 1949.
22. Leck, J. H.: Pressure Measurements in Vacuum Systems. Ch. 2 - Thermoconductivity Gages. Inst. Phys., Chapman and Hall, Ltd. (London), 1957. pp. 33-64.
23. Laurmann, J. A., and Ipsen, D. C.: Use of a Free Molecule Probe in High Speed Rarefied Gas Flow Studies. HE-150-146, Inst. Eng. Res., Univ. Calif., Apr. 30, 1957. (Contract AF-33(616)-2878.)
24. Lowell, Herman H., and Patton, Norman: Response of Homogeneous and Two-Material Laminated Cylinders to Sinusoidal Environmental Temperature Change, with Application to Hot-Wire Anemometry and Thermocouple Pyrometry. NACA TN 3514, 1955.

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TABLE I. - PHYSICAL PROPERTIES OF WIRES USED IN THIS EXPERIMENT

[Wire length = 0.50; wire diam., 2.11×10^{-4} in.; β_w , 2.67×10^{-7} $^{\circ}\text{F}^{-2}$.]

| Probe | Ω_0 , ohms | α_w , $^{\circ}\text{F}^{-1}$ |
|-------|----------------------|---|
| 57 | 42.80 | 1.44×10^{-3} |
| 61 | 54.64 | 1.50×10^{-3} |
| 63 | 51.40 | 1.53×10^{-3} |
| 64 | 54.51 | 1.48×10^{-3} |

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TABLE II. - HEAT TRANSFER FROM CYLINDERS IN TRANSITION FROM SLIP FLOW TO FREE-MOLECULE FLOW

| M | Kn | Re _t | T _t | T _w | Nu _t ^u | Nu _t | Run | M | Kn | Re _t | T _t | T _w | Nu _t ^u | Nu _t | Run |
|--------|--------|-----------------|----------------|----------------|------------------------------|-----------------|-----|--------|--------|-----------------|----------------|----------------|------------------------------|-----------------|-----|
| 0.1039 | 0.2570 | 0.5899 | 76.0 | 150.0 | 0.3422 | 0.3233 | 29 | 0.2156 | 2.078 | 0.1507 | 78.0 | 150.0 | 0.0811 | 0.0715 | 211 |
| | | | | 180.0 | .3464 | .3278 | 30 | | | | | 180.0 | .0794 | .0701 | 212 |
| | | | | 280.0 | .3491 | .3315 | 31 | | | | | 280.0 | .0801 | .0713 | 213 |
| | | | | 380.0 | .3477 | .3310 | 32 | | | | | 380.0 | .0786 | .0702 | 214 |
| | | | | 480.0 | .3392 | .3234 | 33 | | | | | 480.0 | .0705 | .0686 | 215 |
| | | | | 580.0 | .3610 | .3454 | 34 | | | | | 580.0 | .0728 | .0654 | 216 |
| | | | | 580.0 | .3403 | .3246 | 35 | | | | | 180.0 | .0783 | .0690 | 217 |
| 0.1088 | 0.5321 | 0.2977 | 80.0 | 150.0 | 0.2233 | 0.2078 | 239 | 0.2172 | 4.022 | 0.0784 | 79.0 | 150.0 | 0.0271 | 0.0211 | 420 |
| | | | | 180.0 | .2228 | .2076 | 240 | | | | | 180.0 | .0223 | .0168 | 421 |
| | | | | 280.0 | .2254 | .2110 | 241 | | | | | 280.0 | .0316 | .0257 | 422 |
| | | | | 380.0 | .2280 | .2143 | 242 | | | | | 380.0 | .0350 | .0290 | 423 |
| | | | | 480.0 | .2438 | .2302 | 243 | | | | | 480.0 | .0335 | .0278 | 424 |
| | | | | 580.0 | .2558 | .2425 | 244 | | | | | 580.0 | .0414 | .0355 | 425 |
| | | | | 150.0 | .2222 | .2068 | 245 | | | | | 480.0 | .0338 | .0281 | 426 |
| 0.1094 | 1.045 | 0.1524 | 82.0 | 150.0 | 0.1390 | 0.1267 | 50 | 0.2211 | 4.736 | 0.0678 | 78.0 | 150.0 | 0.0430 | 0.0357 | 22 |
| | | | | 180.0 | .1355 | .1236 | 51 | | | | | 180.0 | .0396 | .0327 | 23 |
| | | | | 280.0 | .1500 | .1382 | 52 | | | | | 280.0 | .0390 | .0325 | 24 |
| | | | | 380.0 | .1489 | .1377 | 53 | | | | | 380.0 | .0338 | .0279 | 25 |
| | | | | 480.0 | .1387 | .1283 | 54 | | | | | 480.0 | .0346 | .0269 | 26 |
| | | | | 580.0 | .1576 | .1471 | 55 | | | | | 580.0 | .0356 | .0300 | 27 |
| | | | | 480.0 | .1374 | .1271 | 56 | | | | | 380.0 | .0377 | .0316 | 28 |
| 0.1099 | 2.066 | 0.0776 | 76.0 | 150.0 | 0.0980 | 0.0875 | 302 | 0.2225 | 3.503 | 0.0921 | 78.0 | 150.0 | 0.1210 | 0.1093 | 392 |
| | | | | 180.0 | .0826 | .0730 | 303 | | | | | 180.0 | .0830 | .0734 | 393 |
| | | | | 280.0 | .0859 | .0767 | 304 | | | | | 280.0 | .0636 | .0555 | 394 |
| | | | | 380.0 | .0862 | .0774 | 305 | | | | | 380.0 | .0525 | .0455 | 395 |
| | | | | 480.0 | .0841 | .0758 | 306 | | | | | 480.0 | .0599 | .0527 | 396 |
| | | | | 580.0 | .0830 | .0751 | 307 | | | | | 580.0 | .0566 | .0499 | 397 |
| | | | | 480.0 | .0867 | .0783 | 308 | | | | | 480.0 | .0600 | .0528 | 398 |
| 0.1101 | 0.1074 | 1.491 | 83.0 | 150.0 | 0.3002 | 0.2823 | 176 | 0.3015 | 1.001 | 0.4343 | 80.0 | 150.0 | 0.1513 | 0.1443 | 358 |
| | | | | 180.0 | .3063 | .2887 | 177 | | | | | 180.0 | .1548 | .1421 | 359 |
| | | | | 280.0 | .3330 | .3156 | 178 | | | | | 280.0 | .1589 | .1467 | 360 |
| | | | | 380.0 | .4081 | .3899 | 179 | | | | | 380.0 | .1552 | .1438 | 361 |
| | | | | 480.0 | .4550 | .4367 | 180 | | | | | 480.0 | .1504 | .1396 | 362 |
| | | | | 580.0 | .5205 | .5018 | 181 | | | | | 580.0 | .1611 | .1504 | 363 |
| | | | | 580.0 | .3275 | .3125 | 182 | | | | | 180.0 | .1523 | .1337 | 364 |
| 0.1233 | 3.474 | 0.0517 | 77.0 | 150.0 | 0.1200 | 0.1084 | 113 | 0.3111 | 0.2616 | 1.717 | 87.0 | 150.0 | 0.4113 | 0.3907 | 463 |
| | | | | 180.0 | .0826 | .0730 | 114 | | | | | 180.0 | .4162 | .3959 | 464 |
| | | | | 280.0 | .0604 | .0525 | 115 | | | | | 280.0 | .4163 | .3978 | 465 |
| | | | | 380.0 | .0500 | .0431 | 116 | | | | | 380.0 | .4360 | .4174 | 466 |
| | | | | 480.0 | .0580 | .0509 | 117 | | | | | 480.0 | .4400 | .4222 | 467 |
| | | | | 580.0 | .0562 | .0495 | 118 | | | | | 580.0 | .4436 | .4264 | 468 |
| | | | | 180.0 | .0854 | .0757 | 119 | | | | | 180.0 | .4347 | .4140 | 469 |
| 0.2119 | 0.1049 | 2.927 | 84.0 | 150.0 | 0.6593 | 0.6332 | 281 | 0.3139 | 0.1016 | 4.281 | 84.0 | 150.0 | 0.7191 | 0.6920 | 99 |
| | | | | 180.0 | .6731 | .6473 | 282 | | | | | 180.0 | .7387 | .7118 | 100 |
| | | | | 280.0 | .6962 | .6716 | 283 | | | | | 280.0 | .7553 | .7296 | 101 |
| | | | | 380.0 | .6992 | .6758 | 284 | | | | | 380.0 | .7457 | .7216 | 102 |
| | | | | 480.0 | .6820 | .6600 | 285 | | | | | 480.0 | .7363 | .7135 | 103 |
| | | | | 580.0 | .6850 | .6642 | 286 | | | | | 580.0 | .7347 | .7129 | 104 |
| | | | | 580.0 | .6807 | .6597 | 287 | | | | | 480.0 | .7382 | .7153 | 105 |
| 0.2121 | 0.1076 | 2.853 | 85.0 | 150.0 | 0.6592 | 0.6340 | 78 | 0.3141 | 0.1017 | 4.160 | 86.5 | 147.0 | 0.7171 | 0.6909 | 162 |
| | | | | 179.5 | .6727 | .6478 | 79 | | | | | 179.45 | .7288 | .7030 | 163 |
| | | | | 283.8 | .6889 | .6654 | 80 | | | | | 283.95 | .7485 | .7240 | 164 |
| | | | | 385.0 | .6954 | .6730 | 81 | | | | | 383.95 | .7469 | .7237 | 165 |
| | | | | 486.5 | .6920 | .6708 | 82 | | | | | 486.45 | .7426 | .7206 | 166 |
| | | | | 584.5 | .6920 | .6718 | 83 | | | | | 584.45 | .7407 | .7198 | 167 |
| | | | | 385.0 | .6911 | .6688 | 84 | | | | | 486.45 | .7408 | .7189 | 168 |
| | 0.2609 | 1.177 | 85.0 | 154.9 | 0.3917 | 0.3714 | 323 | 0.3144 | 1.056 | 0.4318 | 82.0 | 150.0 | 0.1574 | 0.1444 | 470 |
| | | | | 187.9 | .3851 | .3654 | 324 | | | | | 180.0 | .1547 | .1420 | 471 |
| | | | | 286.9 | .3855 | .3698 | 325 | | | | | 280.0 | .1586 | .1464 | 472 |
| | | | | 385.9 | .4077 | .3895 | 326 | | | | | 380.0 | .1549 | .1435 | 473 |
| | | | | 483.9 | .4271 | .4093 | 327 | | | | | 480.0 | .1500 | .1392 | 474 |
| | | | | 581.9 | .4315 | .4144 | 328 | | | | | 580.0 | .1607 | .1500 | 475 |
| | | | | 286.9 | .3997 | .3807 | 329 | | | | | 180.0 | .1522 | .1396 | 476 |
| 0.2151 | 1.049 | 0.2971 | 82.0 | 150.0 | 0.1514 | 0.1386 | 141 | 0.3157 | 2.076 | 0.2199 | 76.0 | 180.0 | 0.0794 | 0.0701 | 225 |
| | | | | 180.0 | .1465 | .1341 | 142 | | | | | 280.0 | .0836 | .0746 | 226 |
| | | | | 280.0 | .1588 | .1467 | 143 | | | | | 380.0 | .0858 | .0771 | 227 |
| | | | | 380.0 | .1535 | .1421 | 144 | | | | | 480.0 | .0919 | .0833 | 228 |
| | | | | 480.0 | .1448 | .1342 | 145 | | | | | 580.0 | .0940 | .0857 | 229 |
| | | | | 580.0 | .1593 | .1486 | 146 | | | | | 380.0 | .0915 | .0825 | 230 |
| | | | | 580.0 | .1604 | .1498 | 147 | | | | | | | | |

TABLE II. - Continued. HEAT TRANSFER FROM CYLINDERS IN TRANSITION FROM SLIP FLOW TO FREE-MOLECULE FLOW

| M | Kn | Re _t | T _t | T _w | Nu _t ^h | Nu _t | Run | M | Kn | Re _t | T _t | T _w | Nu _t ^h | Nu _t | Run |
|--------|--------|-----------------|----------------|----------------|------------------------------|-----------------|-----|--------|--------|-----------------|----------------|----------------|------------------------------|-----------------|-----|
| 0.3165 | 0.5267 | 0.8655 | 84.0 | 150.0 | 0.2897 | 0.2720 | 441 | 0.4246 | 2.110 | 0.2880 | 81.0 | 150.0 | 0.1016 | 0.0910 | 253 |
| | | | | 180.0 | .2836 | .2665 | 442 | | | | | 180.0 | .0955 | .0854 | 254 |
| | | | | 280.0 | .2789 | .2629 | 443 | | | | | 280.0 | .0933 | .0838 | 255 |
| | | | | 380.0 | .2755 | .2604 | 444 | | | | | 380.0 | .0900 | .0812 | 256 |
| | | | | 480.0 | .2765 | .2621 | 445 | | | | | 480.0 | .0921 | .0836 | 257 |
| | | | | 580.0 | .2782 | .2644 | 446 | | | | | 580.0 | .0871 | .0791 | 258 |
| | | | | 380.0 | .2253 | .2153 | 447 | | | | | 380.0 | .0923 | .0833 | 259 |
| 0.3278 | 3.517 | 0.1346 | 77.5 | 150.0 | 0.1204 | 0.1088 | 477 | 0.4283 | 3.495 | 0.1756 | 78.0 | 150.0 | 0.1241 | 0.1123 | 120 |
| | | | | 180.0 | .0841 | .0745 | 478 | | | | | 180.0 | .0852 | .0755 | 121 |
| | | | | 280.0 | .0596 | .0518 | 479 | | | | | 280.0 | .0636 | .0555 | 122 |
| | | | | 380.0 | .0525 | .0454 | 480 | | | | | 380.0 | .0561 | .0488 | 122 |
| | | | | 480.0 | .0605 | .0533 | 481 | | | | | 480.0 | .0623 | .0550 | 124 |
| | | | | 580.0 | .0576 | .0508 | 482 | | | | | 580.0 | .0581 | .0515 | 125 |
| | | | | 280.0 | .0582 | .0512 | 483 | | | | | 280.0 | .0636 | .0554 | 126 |
| 0.3281 | 4.720 | 0.1004 | 77.0 | 150.0 | 0.0328 | 0.0263 | 267 | 0.5085 | 1.053 | 0.6855 | 82.0 | 150.0 | 0.2136 | 0.1984 | 246 |
| | | | | 180.0 | .0349 | .0284 | 268 | | | | | 180.0 | .2015 | .1871 | 247 |
| | | | | 280.0 | .0373 | .0308 | 269 | | | | | 280.0 | .1892 | .1759 | 248 |
| | | | | 380.0 | .0373 | .0311 | 270 | | | | | 380.0 | .1733 | .1612 | 249 |
| | | | | 480.0 | .0372 | .0313 | 271 | | | | | 480.0 | .1696 | .1582 | 250 |
| | | | | 580.0 | .0368 | .0311 | 272 | | | | | 580.0 | .1813 | .1700 | 251 |
| | | | | 380.0 | .0373 | .0311 | 273 | | | | | 580.0 | .1816 | .1703 | 252 |
| 0.3287 | 4.033 | 0.1175 | 80.0 | 150.0 | 0.0252 | 0.0194 | 456 | 1.045 | 0.6920 | 78.0 | 150.0 | 0.1739 | 0.1603 | 85 | |
| | | | | 180.0 | .0320 | .0256 | 457 | | | | 180.0 | .1738 | .1604 | 86 | |
| | | | | 280.0 | .0376 | .0311 | 458 | | | | 280.0 | .1755 | .1628 | 87 | |
| | | | | 380.0 | .0398 | .0335 | 459 | | | | 380.0 | .1651 | .1534 | 88 | |
| | | | | 480.0 | .0408 | .0347 | 460 | | | | 480.0 | .1637 | .1526 | 89 | |
| | | | | 580.0 | .0425 | .0365 | 461 | | | | 580.0 | .1765 | .1654 | 90 | |
| | | | | 480.0 | .0406 | .0345 | 462 | | | | 580.0 | .1768 | .1657 | 91 | |
| 0.4051 | 0.2575 | 2.249 | 86.0 | 155.0 | 0.5098 | 0.4869 | 1 | 0.5114 | 0.1078 | 6.7323 | 83.0 | 385.0 | 0.8519 | 0.8272 | 235 |
| | | | | 188.0 | .4923 | .4703 | 2 | | | | | 486.5 | .8447 | .8213 | 236 |
| | | | | 287.0 | .4655 | .4452 | 3 | | | | | 584.5 | .8541 | .8316 | 237 |
| | | | | 386.0 | .4709 | .4516 | 4 | | | | | | | | |
| | | | | 484.0 | .4630 | .4447 | 5 | | | | | | | | |
| | | | | 582.0 | .4597 | .4422 | 6 | | | | | | | | |
| | | | | 582.0 | .4621 | .4446 | 7 | | | | | | | | |
| 0.4125 | 1.040 | 0.5696 | 75.0 | 150.0 | 0.1579 | 0.1448 | 183 | 0.5151 | 0.2780 | 2.6929 | 86.0 | 147.0 | 0.4625 | 0.4413 | 330 |
| | | | | 180.0 | .1658 | .1526 | 184 | | | | | 179.5 | .4693 | .4485 | 331 |
| | | | | 280.0 | .1600 | .1478 | 185 | | | | | 284.0 | .4832 | .4634 | 332 |
| | | | | 380.0 | .1580 | .1464 | 186 | | | | | 385.0 | .4768 | .4582 | 333 |
| | | | | 480.0 | .1555 | .1445 | 187 | | | | | 486.5 | .4668 | .4492 | 334 |
| | | | | 580.0 | .1677 | .1568 | 188 | | | | | 584.5 | .4655 | .4464 | 335 |
| | | | | 380.0 | .1568 | .1453 | 189 | | | | | 284.0 | .4796 | .4599 | 336 |
| 0.4146 | 0.1074 | 5.524 | 82.0 | 147.0 | 0.7358 | 0.7093 | 344 | 0.5165 | 0.2643 | 2.7663 | 86.0 | 155.0 | 0.4896 | 0.4672 | 36 |
| | | | | 179.4 | .7627 | .7362 | 345 | | | | | 188.0 | .4818 | .4601 | 37 |
| | | | | 283.9 | .8072 | .7817 | 346 | | | | | 287.0 | .4656 | .4454 | 38 |
| | | | | 384.9 | .8009 | .7769 | 347 | | | | | 386.0 | .4730 | .4538 | 39 |
| | | | | 486.4 | .8063 | .7834 | 348 | | | | | 484.0 | .4670 | .4487 | 40 |
| | | | | 584.4 | .8149 | .7929 | 349 | | | | | 581.0 | .4622 | .4448 | 41 |
| | | | | 584.4 | .8068 | .7849 | 350 | | | | | 484.0 | .4641 | .4459 | 42 |
| 0.4147 | 0.5273 | 1.125 | 83.0 | 150.0 | 0.2974 | 0.2794 | 57 | 0.5231 | 2.095 | 0.3548 | 77.0 | 150.0 | 0.0855 | 0.0756 | 148 |
| | | | | 180.0 | .2956 | .2781 | 58 | | | | | 180.0 | .0860 | .0768 | 149 |
| | | | | 280.0 | .2955 | .2790 | 59 | | | | | 280.0 | .1012 | .0912 | 150 |
| | | | | 380.0 | .2879 | .2724 | 60 | | | | | 380.0 | .1003 | .0909 | 151 |
| | | | | 480.0 | .2886 | .2738 | 61 | | | | | 480.0 | .0955 | .0867 | 152 |
| | | | | 580.0 | .2869 | .2728 | 62 | | | | | 580.0 | .0956 | .0871 | 153 |
| | | | | 380.0 | .2886 | .2731 | 63 | | | | | 380.0 | .1446 | .1335 | 154 |
| 0.4165 | 0.2714 | 2.190 | 87.5 | 147.0 | 0.4373 | 0.4168 | 372 | 0.5248 | 4.699 | 0.1588 | 75.5 | 150.0 | 0.0460 | 0.0384 | 427 |
| | | | | 179.4 | .4454 | .4251 | 373 | | | | | 180.0 | .0455 | .0381 | 428 |
| | | | | 283.9 | .4534 | .4343 | 374 | | | | | 280.0 | .0458 | .0388 | 429 |
| | | | | 384.9 | .4484 | .4304 | 375 | | | | | 380.0 | .0458 | .0390 | 430 |
| | | | | 486.4 | .4410 | .4240 | 376 | | | | | 480.0 | .0451 | .0386 | 431 |
| | | | | 584.4 | .4388 | .4226 | 377 | | | | | 580.0 | .0442 | .0381 | 432 |
| | | | | | | | | | | | | 380.0 | .0456 | .0388 | 433 |
| 0.4186 | 4.706 | 0.1276 | 77.0 | 150.0 | 0.0314 | 0.0249 | 406 | 0.5250 | 0.5268 | 1.4112 | 84.0 | 150.0 | 0.3090 | 0.2909 | 309 |
| | | | | 180.0 | .0360 | .0293 | 407 | | | | | 180.0 | .3168 | .2989 | 310 |
| | | | | 280.0 | .0428 | .0353 | 408 | | | | | 280.0 | .3066 | .2899 | 311 |
| | | | | 380.0 | .0423 | .0358 | 409 | | | | | 380.0 | .3063 | .2905 | 312 |
| | | | | 480.0 | .0422 | .0359 | 410 | | | | | 480.0 | .3158 | .2986 | 313 |
| | | | | 580.0 | .0417 | .0357 | 411 | | | | | 580.0 | .3146 | .2900 | 314 |
| | | | | 380.0 | .0445 | .0378 | 412 | | | | | 380.0 | .3007 | .2850 | 315 |

TABLE II. - Continued. HEAT TRANSFER FROM CYLINDERS IN TRANSITION FROM SLIP FLOW TO FREE-MOLECULE FLOW

| M | Kn | Re _t | T _t | T _w | Nu _t ¹ | Nu _t | Run | M | Kn | Re _t | T _t | T _w | Nu _t ¹ | Nu _t | Run |
|--------|--------|-----------------|----------------|----------------|------------------------------|-----------------|-----|--------|--------|-----------------|----------------|----------------|------------------------------|-----------------|-----|
| 0.5279 | 3.484 | 0.2151 | 78.0 | 150.0 | 0.1235 | 0.1117 | 399 | 0.7063 | 2.091 | 0.4700 | 78.0 | 150.0 | 0.1023 | 0.0916 | 413 |
| | | | | 180.0 | .0900 | .0800 | 400 | | | | | 180.0 | .1058 | .0951 | 414 |
| | | | | 280.0 | .0660 | .0578 | 401 | | | | | 280.0 | .1064 | .0963 | 415 |
| | | | | 380.0 | .0591 | .0516 | 402 | | | | | 380.0 | .1020 | .0925 | 416 |
| | | | | 480.0 | .0633 | .0559 | 403 | | | | | 480.0 | .0999 | .0909 | 417 |
| | | | | 580.0 | .0593 | .0524 | 404 | | | | | 580.0 | .0977 | .0892 | 418 |
| | | | | 380.0 | .0591 | .0517 | 405 | | | | | 280.0 | .1028 | .0928 | 419 |
| 0.6036 | 2.062 | 0.4125 | 77.5 | 150.0 | 0.0912 | 0.0811 | 365 | 2.102 | 0.4667 | 81.0 | 150.0 | 0.1171 | 0.1056 | 190 | |
| | | | | 180.0 | .0965 | .0863 | 366 | | | | 180.0 | .1162 | .1050 | 191 | |
| | | | | 280.0 | .0941 | .0845 | 367 | | | | 280.0 | .1113 | .1009 | 192 | |
| | | | | 380.0 | .0958 | .0866 | 368 | | | | 380.0 | .1050 | .0953 | 193 | |
| | | | | 480.0 | .0943 | .0855 | 369 | | | | 480.0 | .1019 | .0928 | 194 | |
| | | | | 580.0 | .0927 | .0844 | 370 | | | | 580.0 | .1002 | .0892 | 195 | |
| | | | | 480.0 | .0958 | .0870 | 371 | | | | 180.0 | .1162 | .1048 | 196 | |
| 0.6079 | 0.1086 | 7.854 | 85.5 | 150.0 | 0.8520 | 0.8235 | 106 | 0.7096 | 3.476 | 0.2839 | 78.0 | 150.0 | 0.1290 | 0.1170 | 434 |
| | | | | 180.0 | .8720 | .8438 | 107 | | | | | 180.0 | .0911 | .0811 | 435 |
| | | | | 280.0 | .8727 | .8463 | 108 | | | | | 280.0 | .0729 | .0643 | 436 |
| | | | | 380.0 | .8379 | .8136 | 109 | | | | | 380.0 | .0727 | .0645 | 437 |
| | | | | 480.0 | .8434 | .8202 | 110 | | | | | 480.0 | .0747 | .0668 | 438 |
| | | | | 580.0 | .8300 | .8080 | 111 | | | | | 580.0 | .0689 | .0616 | 439 |
| | | | | 480.0 | .8240 | .8011 | 112 | | | | | 380.0 | .0721 | .0640 | 440 |
| 0.6146 | 1.054 | 0.8187 | 82.0 | 150.0 | 0.1607 | 0.1475 | 295 | 4.122 | 0.2089 | 78.0 | 150.0 | 0.0459 | 0.0383 | 64 | |
| | | | | 180.0 | .1667 | .1534 | 296 | | | | 180.0 | .0490 | .0413 | 65 | |
| | | | | 280.0 | .1687 | .1561 | 297 | | | | 280.0 | .0525 | .0449 | 66 | |
| | | | | 380.0 | .1666 | .1547 | 298 | | | | 380.0 | .0527 | .0455 | 67 | |
| | | | | 480.0 | .1650 | .1537 | 299 | | | | 480.0 | .0513 | .0445 | 68 | |
| | | | | 580.0 | .1827 | .1713 | 300 | | | | 580.0 | .0514 | .0449 | 69 | |
| | | | | 380.0 | .1666 | .1547 | 301 | | | | 280.0 | .0524 | .0449 | 70 | |
| 0.6163 | 4.052 | 0.2135 | 82.0 | 150.0 | 0.0417 | 0.0344 | 169 | 0.7131 | 4.000 | 0.2478 | 78.0 | 150.0 | 0.0469 | 0.0392 | 449 |
| | | | | 180.0 | .0465 | .0390 | 170 | | | | | 180.0 | .0521 | .0442 | 450 |
| | | | | 280.0 | .0522 | .0447 | 171 | | | | | 280.0 | .0566 | .0489 | 451 |
| | | | | 380.0 | .0528 | .0456 | 172 | | | | | 380.0 | .0564 | .0491 | 452 |
| | | | | 480.0 | .0519 | .0451 | 173 | | | | | 480.0 | .0557 | .0487 | 453 |
| | | | | 580.0 | .0519 | .0453 | 174 | | | | | 580.0 | .0559 | .0491 | 454 |
| | | | | 180.0 | .0489 | .0412 | 175 | | | | | 280.0 | .0563 | .0489 | 455 |
| 0.6222 | 4.722 | 0.1852 | 78.0 | 150.0 | 0.0504 | 0.0425 | 204 | 0.7177 | 1.153 | 0.9453 | 82.0 | 150.0 | 0.1699 | 0.1563 | 378 |
| | | | | 180.0 | .0506 | .0428 | 205 | | | | | 180.0 | .1665 | .1534 | 379 |
| | | | | 280.0 | .0503 | .0430 | 206 | | | | | 280.0 | .1732 | .1604 | 380 |
| | | | | 380.0 | .0501 | .0430 | 207 | | | | | 380.0 | .1717 | .1597 | 381 |
| | | | | 480.0 | .0490 | .0423 | 208 | | | | | 480.0 | .1689 | .1574 | 382 |
| | | | | 580.0 | .0484 | .0420 | 209 | | | | | 580.0 | .1871 | .1756 | 383 |
| | | | | 480.0 | .0516 | .0448 | 210 | | | | | 480.0 | .1688 | .1573 | 384 |
| 0.6240 | 3.465 | 0.2522 | 76.0 | 150.0 | 0.1230 | 0.1113 | 15 | 0.7722 | 1.161 | 1.002 | 81.0 | 150.0 | 0.1793 | 0.1653 | 218 |
| | | | | 180.0 | .0882 | .0783 | 16 | | | | | 180.0 | .1766 | .1630 | 219 |
| | | | | 280.0 | .0694 | .0610 | 17 | | | | | 280.0 | .1802 | .1672 | 220 |
| | | | | 380.0 | .0679 | .0599 | 18 | | | | | 380.0 | .1785 | .1662 | 221 |
| | | | | 480.0 | .0654 | .0579 | 19 | | | | | 480.0 | .1751 | .1635 | 222 |
| | | | | 580.0 | .0643 | .0571 | 20 | | | | | 580.0 | .1910 | .1794 | 223 |
| | | | | 380.0 | .0670 | .0591 | 21 | | | | | 380.0 | .1790 | .1667 | 224 |
| 0.6851 | 0.1095 | 8.684 | 88.0 | 147.0 | 0.8845 | 0.8555 | 274 | 0.7882 | 0.085 | 9.965 | 84.0 | 147.0 | 0.8773 | 0.8483 | 155 |
| | | | | 179.5 | .9093 | .8806 | 275 | | | | | 179.5 | .9702 | .9405 | 156 |
| | | | | 284.0 | .9295 | .9093 | 276 | | | | | 284.0 | .9780 | .9500 | 157 |
| | | | | 385.0 | .9122 | .8868 | 277 | | | | | 375.0 | .9567 | .9306 | 158 |
| | | | | 486.5 | .9138 | .8896 | 278 | | | | | 486.5 | .9511 | .9263 | 159 |
| | | | | 584.5 | .8898 | .8670 | 279 | | | | | 584.5 | .9631 | .9393 | 160 |
| | | | | 584.5 | .8898 | .8670 | 280 | | | | | 284.0 | .9934 | .9652 | 161 |
| 0.6871 | 0.2663 | 3.583 | 86.0 | 155.0 | 0.5287 | 0.5055 | 127 | 0.7938 | 2.11 | 0.5171 | 77.5 | 150.0 | 0.0977 | 0.0872 | 92 |
| | | | | 188.0 | .5209 | .4984 | 128 | | | | | 180.0 | .1013 | .0908 | 93 |
| | | | | 287.0 | .5036 | .4827 | 129 | | | | | 280.0 | .1044 | .0944 | 94 |
| | | | | 386.0 | .5066 | .4867 | 130 | | | | | 380.0 | .1027 | .0932 | 95 |
| | | | | 484.0 | .4979 | .4791 | 131 | | | | | 480.0 | .1008 | .0918 | 96 |
| | | | | 582.0 | .4910 | .4731 | 132 | | | | | 580.0 | .0995 | .0909 | 97 |
| | | | | 386.0 | .5015 | .4817 | 133 | | | | | 280.0 | .1040 | .0940 | 98 |
| 0.7042 | 0.5416 | 1.802 | 86.0 | 147.0 | 0.3034 | 0.2861 | 351 | 0.7975 | 4.05 | 0.2330 | 77.0 | 150.0 | 0.0567 | 0.0482 | 337 |
| | | | | 189.5 | .3080 | .2910 | 352 | | | | | 180.0 | .0564 | .0482 | 338 |
| | | | | 284.0 | .3173 | .3011 | 353 | | | | | 280.0 | .0565 | .0488 | 339 |
| | | | | 385.0 | .3102 | .2951 | 354 | | | | | 380.0 | .0556 | .0483 | 340 |
| | | | | 486.5 | .3030 | .2887 | 355 | | | | | 480.0 | .0568 | .0497 | 341 |
| | | | | 584.5 | .3131 | .2993 | 356 | | | | | 580.0 | .0536 | .0469 | 342 |
| | | | | 486.5 | .3041 | .2898 | 357 | | | | | 280.0 | .0560 | .0483 | 343 |

| M | Kn | Re _t | T _t | T _w | Nu _t | Nu _t | Run | M | Kn | Re _t | T _t | T _w | Nu _t | Nu _t | Run | | | | | | | |
|--------|--------|-----------------|----------------|----------------|-----------------|-----------------|------|--------|--------|-----------------|----------------|----------------|-----------------|-----------------|------|--------|--------|--------|------|-------|--------|--------|
| 0.7979 | 3.506 | 0.3127 | 78.0 | 150.0 | 0.1389 | 0.1264 | 288 | 0.8674 | 0.2641 | 4.446 | 87.0 | 155.0 | 0.5705 | 0.5465 | 71 | | | | | | | |
| | | | | 180.0 | .0975 | .0871 | 289 | | | | | 188.0 | .5579 | .5347 | 72 | | | | | | | |
| | | | | 280.0 | .0784 | .0695 | 290 | | | | | 287.0 | .5294 | .5080 | 73 | | | | | | | |
| | | | | 380.0 | .0757 | .0674 | 291 | | | | | 386.0 | .5260 | .5057 | 74 | | | | | | | |
| | | | | 480.0 | .0749 | .0669 | 292 | | | | | 484.0 | .5171 | .4980 | 75 | | | | | | | |
| | | | | 580.0 | .0741 | .0665 | 293 | | | | | 582.0 | .5463 | .5275 | 76 | | | | | | | |
| | | | | 280.0 | .0782 | .0693 | 294 | | | | | 582.0 | .5437 | .5249 | 77 | | | | | | | |
| | | | | 0.8104 | 0.2765 | 4.008 | 84.5 | | | | | 147.0 | 0.5353 | 0.5127 | 8 | 0.8733 | 0.1096 | 10.781 | 87.0 | 147.0 | 0.9865 | 0.9559 |
| 179.5 | .5386 | .5163 | 9 | | | | | 179.5 | .9928 | .9628 | 198 | | | | | | | | | | | |
| 184.0 | .5315 | .5179 | 10 | | | | | 284.0 | 1.0074 | .9792 | 199 | | | | | | | | | | | |
| 385.0 | .5201 | .5120 | 11 | | | | | 385.0 | 1.0002 | .9735 | 200 | | | | | | | | | | | |
| 486.5 | .5409 | .5018 | 12 | | | | | 486.5 | .9882 | .9631 | 201 | | | | | | | | | | | |
| 584.5 | .5201 | .5231 | 13 | | | | | 584.5 | 1.0307 | 1.0061 | 202 | | | | | | | | | | | |
| 486.5 | .5201 | .5018 | 14 | | | | | 284.0 | 1.0047 | .9764 | 203 | | | | | | | | | | | |
| 0.8268 | 4.006 | 0.2823 | 79.0 | | | | | 150.0 | 0.0410 | 0.0337 | 385 | 0.8863 | 3.502 | 0.3432 | 78.0 | | | | | 150.0 | 0.1499 | 0.1369 |
| | | | | 180.0 | .0644 | .0561 | 386 | 180.0 | .1046 | .0938 | 261 | | | | | | | | | | | |
| | | | | 280.0 | .0652 | .0561 | 387 | 280.0 | .0818 | .0726 | 262 | | | | | | | | | | | |
| | | | | 380.0 | .0643 | .0569 | 388 | 380.0 | .0801 | .0715 | 263 | | | | | | | | | | | |
| | | | | 480.0 | .0626 | .0565 | 389 | 480.0 | .0794 | .0712 | 264 | | | | | | | | | | | |
| | | | | 580.0 | .0623 | .0554 | 390 | 580.0 | .0781 | .0703 | 265 | | | | | | | | | | | |
| | | | | 380.0 | .0639 | .0561 | 391 | 380.0 | .0782 | .0697 | 266 | | | | | | | | | | | |
| | | | | 0.8363 | 2.027 | 0.5639 | 78.0 | 150.0 | 0.1110 | 0.0998 | 316 | | | | | 0.8871 | 4.028 | 0.2991 | 75.0 | 150.0 | 0.0459 | 0.0382 |
| 180.0 | .1175 | .1062 | 317 | | | | | 180.0 | .0678 | .0588 | 135 | | | | | | | | | | | |
| 280.0 | .1123 | .1020 | 318 | | | | | 280.0 | .0702 | .0616 | 136 | | | | | | | | | | | |
| 380.0 | .1108 | .1010 | 319 | | | | | 380.0 | .0690 | .0609 | 137 | | | | | | | | | | | |
| 480.0 | .1079 | .0987 | 320 | | | | | 480.0 | .0667 | .0590 | 138 | | | | | | | | | | | |
| 580.0 | .1062 | .0974 | 321 | | | | | 580.0 | .0660 | .0587 | 139 | | | | | | | | | | | |
| 280.0 | .1115 | .1012 | 322 | | | | | 280.0 | .0686 | .0601 | 140 | | | | | | | | | | | |
| 0.8668 | 0.5417 | 2.166 | 87.5 | | | | | 147.0 | 0.3323 | 0.3143 | 43 | | | | | | | | | | | |
| | | | | 179.5 | .3371 | .3194 | 44 | | | | | | | | | | | | | | | |
| | | | | 284.0 | .3418 | .3250 | 45 | | | | | | | | | | | | | | | |
| | | | | 385.0 | .3380 | .3223 | 46 | | | | | | | | | | | | | | | |
| | | | | 486.5 | .3339 | .3190 | 47 | | | | | | | | | | | | | | | |
| | | | | 484.5 | .3323 | .3181 | 48 | | | | | | | | | | | | | | | |
| | | | | 385.0 | .3385 | .3227 | 49 | | | | | | | | | | | | | | | |

| M | Kn | Re _t | T _t | T _w | Nu _t | Nu _t | Run | M | Kn | Re _t | T _t | T _w | Nu _t | Nu _t | Run | | | | | | | |
|--------|--------|-----------------|----------------|----------------|-----------------|-----------------|------|--------|--------|-----------------|----------------|----------------|-----------------|-----------------|------|--------|--------|--------|------|-------|--------|--------|
| 0.7979 | 3.506 | 0.3127 | 78.0 | 150.0 | 0.1389 | 0.1264 | 288 | 0.8674 | 0.2641 | 4.446 | 87.0 | 155.0 | 0.5705 | 0.5465 | 71 | | | | | | | |
| | | | | 180.0 | .0975 | .0871 | 289 | | | | | 188.0 | .5579 | .5347 | 72 | | | | | | | |
| | | | | 280.0 | .0784 | .0695 | 290 | | | | | 287.0 | .5294 | .5080 | 73 | | | | | | | |
| | | | | 380.0 | .0757 | .0674 | 291 | | | | | 386.0 | .5260 | .5057 | 74 | | | | | | | |
| | | | | 480.0 | .0749 | .0669 | 292 | | | | | 484.0 | .5171 | .4980 | 75 | | | | | | | |
| | | | | 580.0 | .0741 | .0665 | 293 | | | | | 582.0 | .5463 | .5275 | 76 | | | | | | | |
| | | | | 280.0 | .0782 | .0693 | 294 | | | | | 582.0 | .5437 | .5249 | 77 | | | | | | | |
| | | | | 0.8104 | 0.2765 | 4.008 | 84.5 | | | | | 147.0 | 0.5353 | 0.5127 | 8 | 0.8733 | 0.1096 | 10.781 | 87.0 | 147.0 | 0.9865 | 0.9559 |
| 179.5 | .5386 | .5163 | 9 | | | | | 179.5 | .9928 | .9628 | 198 | | | | | | | | | | | |
| 184.0 | .5315 | .5179 | 10 | | | | | 284.0 | 1.0074 | .9792 | 199 | | | | | | | | | | | |
| 385.0 | .5201 | .5120 | 11 | | | | | 385.0 | 1.0002 | .9735 | 200 | | | | | | | | | | | |
| 486.5 | .5409 | .5018 | 12 | | | | | 486.5 | .9882 | .9631 | 201 | | | | | | | | | | | |
| 584.5 | .5201 | .5231 | 13 | | | | | 584.5 | 1.0307 | 1.0061 | 202 | | | | | | | | | | | |
| 486.5 | .5201 | .5018 | 14 | | | | | 284.0 | 1.0047 | .9764 | 203 | | | | | | | | | | | |
| 0.8268 | 4.006 | 0.2823 | 79.0 | | | | | 150.0 | 0.0410 | 0.0337 | 385 | 0.8863 | 3.502 | 0.3432 | 78.0 | | | | | 150.0 | 0.1499 | 0.1369 |
| | | | | 180.0 | .0644 | .0561 | 386 | 180.0 | .1046 | .0938 | 261 | | | | | | | | | | | |
| | | | | 280.0 | .0652 | .0561 | 387 | 280.0 | .0818 | .0726 | 262 | | | | | | | | | | | |
| | | | | 380.0 | .0643 | .0569 | 388 | 380.0 | .0801 | .0715 | 263 | | | | | | | | | | | |
| | | | | 480.0 | .0626 | .0565 | 389 | 480.0 | .0794 | .0712 | 264 | | | | | | | | | | | |
| | | | | 580.0 | .0623 | .0554 | 390 | 580.0 | .0781 | .0703 | 265 | | | | | | | | | | | |
| | | | | 380.0 | .0639 | .0561 | 391 | 380.0 | .0782 | .0697 | 266 | | | | | | | | | | | |
| | | | | 0.8363 | 2.027 | 0.5639 | 78.0 | 150.0 | 0.1110 | 0.0998 | 316 | | | | | 0.8871 | 4.028 | 0.2991 | 75.0 | 150.0 | 0.0459 | 0.0382 |
| 180.0 | .1175 | .1062 | 317 | | | | | 180.0 | .0678 | .0588 | 135 | | | | | | | | | | | |
| 280.0 | .1123 | .1020 | 318 | | | | | 280.0 | .0702 | .0616 | 136 | | | | | | | | | | | |
| 380.0 | .1108 | .1010 | 319 | | | | | 380.0 | .0690 | .0609 | 137 | | | | | | | | | | | |
| 480.0 | .1079 | .0987 | 320 | | | | | 480.0 | .0667 | .0590 | 138 | | | | | | | | | | | |
| 580.0 | .1062 | .0974 | 321 | | | | | 580.0 | .0660 | .0587 | 139 | | | | | | | | | | | |
| 280.0 | .1115 | .1012 | 322 | | | | | 280.0 | .0686 | .0601 | 140 | | | | | | | | | | | |
| 0.8668 | 0.5417 | 2.166 | 87.5 | | | | | 147.0 | 0.3323 | 0.3143 | 43 | | | | | | | | | | | |
| | | | | 179.5 | .3371 | .3194 | 44 | | | | | | | | | | | | | | | |
| | | | | 284.0 | .3418 | .3250 | 45 | | | | | | | | | | | | | | | |
| | | | | 385.0 | .3380 | .3223 | 46 | | | | | | | | | | | | | | | |
| | | | | 486.5 | .3339 | .3190 | 47 | | | | | | | | | | | | | | | |
| | | | | 484.5 | .3323 | .3181 | 48 | | | | | | | | | | | | | | | |
| | | | | 385.0 | .3385 | .3227 | 49 | | | | | | | | | | | | | | | |

TABLE III. - DIMENSIONLESS FUNCTIONS

| Mach number, M | s | g(s) | f(s) | $\frac{T_{e,\infty}}{T_t}$ |
|----------------------|-------|----------|----------|----------------------------|
| 0.05 | 0.042 | 9.43323 | 9.44172 | 1.0004 |
| .10 | .084 | 9.45858 | 9.49210 | 1.0015 |
| .15 | .126 | 9.49908 | 9.57430 | 1.0034 |
| .20 | .167 | 9.55632 | 9.68872 | 1.0058 |
| .25 | .209 | 9.63015 | 9.83834 | 1.0091 |
| .30 | .251 | 9.71958 | 10.02114 | 1.0128 |
| .35 | .293 | 9.82485 | 10.23799 | 1.0171 |
| .40 | .335 | 9.94602 | 10.48942 | 1.0219 |
| .45 | .377 | 10.08321 | 10.77637 | 1.0272 |
| .50 | .418 | 10.22958 | 11.08815 | 1.0323 |
| .55 | .460 | 10.39665 | 11.44568 | 1.0381 |
| .57 | .477 | 10.46736 | 11.59930 | 1.0405 |
| .59 | .494 | 10.54086 | 11.75943 | 1.0430 |
| .61 | .511 | 10.61694 | 11.92605 | 1.0455 |
| .63 | .527 | 10.69080 | 12.08838 | 1.0476 |
| .65 | .544 | 10.76952 | 12.26456 | 1.0501 |
| .67 | .561 | 10.85100 | 12.44746 | 1.0526 |
| .69 | .578 | 10.93728 | 12.63973 | 1.0552 |
| .71 | .594 | 11.01711 | 12.82192 | 1.0572 |
| .73 | .611 | 11.10813 | 13.02687 | 1.0598 |
| .75 | .628 | 11.19696 | 13.23293 | 1.0623 |
| .77 | .644 | 11.28270 | 13.43252 | 1.0643 |
| .79 | .661 | 11.37807 | 13.65378 | 1.0668 |
| .80 | .670 | 11.42904 | 13.77319 | 1.0683 |
| .85 | .711 | 11.66832 | 14.33964 | 1.0737 |
| .90 | .753 | 11.92563 | 14.96017 | 1.0796 |
| .95 | .795 | 12.19029 | 15.61611 | 1.0852 |
| 1.00 | .837 | 12.46626 | 16.31451 | 1.0905 |

TABLE IV. - MEASURED RECOVERY TEMPERATURES

| T_e/T_t | $T_e/T_{e,\infty}$ | M | Kn |
|--|--|--|---|
| 0.9839 .9826 | 0.9615 .9511 | 0.4138 .5080 | 1.052 1.051 |
| 1.000 .9982 .9938 .9894 .9850 .9850 .9901 .9863 .9750 | 0.9982 .9916 .9800 .9660 .9520 .9421 .9367 .9235 .9058 | 0.1097 .2162 .3160 .4231 .5219 .6131 .7041 .7935 .8682 | 2.078 2.082 2.088 2.102 2.103 2.092 2.096 2.117 2.113 |
| 0.9938 .9976 .9964 .9989 .9995 .9989 1.007 1.011 1.019 | 0.9915 .9906 .9817 .9746 .9654 .9542 .9526 .9461 .9453 | 0.1214 .2221 .3258 .4299 .5269 .6231 .7059 .7979 .8846 | 3.487 3.514 3.527 3.507 3.492 3.501 3.486 3.519 3.511 |
| 0.9912 .9969 .9944 1.010 1.019 1.029 1.042 1.054 | 0.9845 .9820 .9707 .9754 .9738 .9727 .9723 .9775 | 0.2170 .3291 .4154 .5306 .6157 .7134 .8261 .8887 | 4.007 4.010 4.013 3.999 4.015 4.000 3.992 4.041 |
| 0.9949 1.009 1.017 1.029 1.040 1.049 1.056 1.067 | 0.9881 .9939 .9931 .9937 .9933 .9921 .9889 .9901 | 0.2207 .3274 .4170 .5260 .6210 .7083 .7953 .8836 | 4.710 4.703 4.689 4.694 4.697 4.698 4.682 4.774 |

TABLE V. - ACCOMMODATION COEFFICIENT CALCULATED USING
 MEASURED NUSSELT NUMBERS AND FREE-MOLECULE-
 FLOW PREDICTION (EQ. (13))

| M | $Nu_{t,meas}$ | Kn | α_{calc} |
|----------------------|---------------|--------|-----------------|
| 0.5248 | 0.03808 | 4.6994 | 0.545 |
| .7975 | .04694 | 4.7051 | .655 |
| .3281 | .03110 | 4.7199 | .482 |
| .8901 | .05322 | 4.8001 | .776 |
| .2211 | .03001 | 4.7359 | .472 |
| .7095 | .04486 | 4.7225 | .638 |
| .4186 | .03565 | 4.7058 | .545 |
| .6222 | .04196 | 4.7218 | .616 |
| .5311 | .04203 | 4.0135 | .532 |
| .8268 | .05514 | 4.0064 | .654 |
| .2172 | .03550 | 4.0223 | .496 |
| .8871 | .05866 | 4.0283 | .706 |
| .4270 | .03800 | 4.0050 | .488 |
| .7131 | .04900 | 4.0001 | .597 |
| .3287 | .03650 | 4.0330 | .482 |
| .6163 | .04530 | 4.0520 | .572 |
| $\alpha_{av} = 0.57$ | | | |

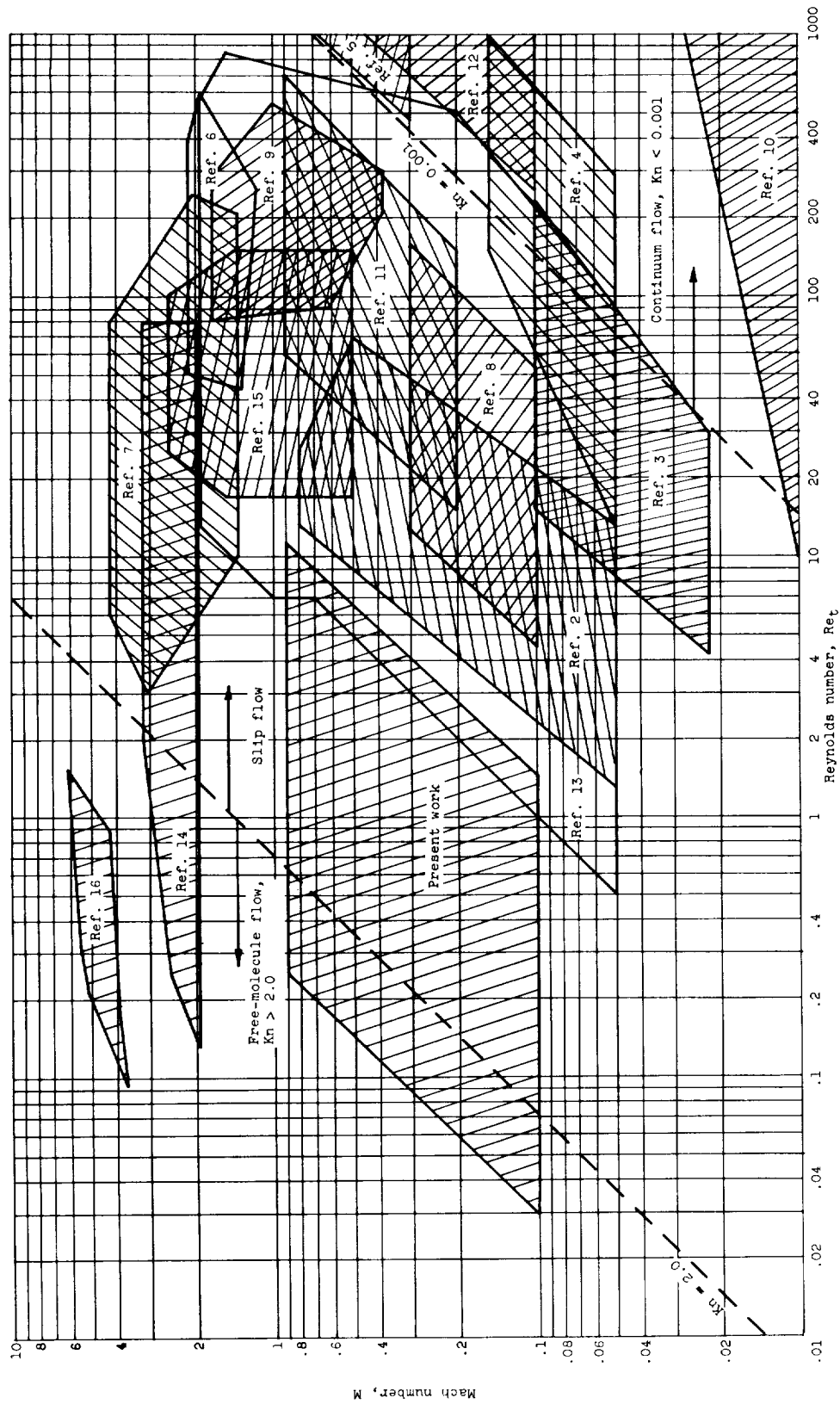


Figure 1. - Comparison of Mach and Reynolds number ranges of heat-transfer experiments for previous and present work.

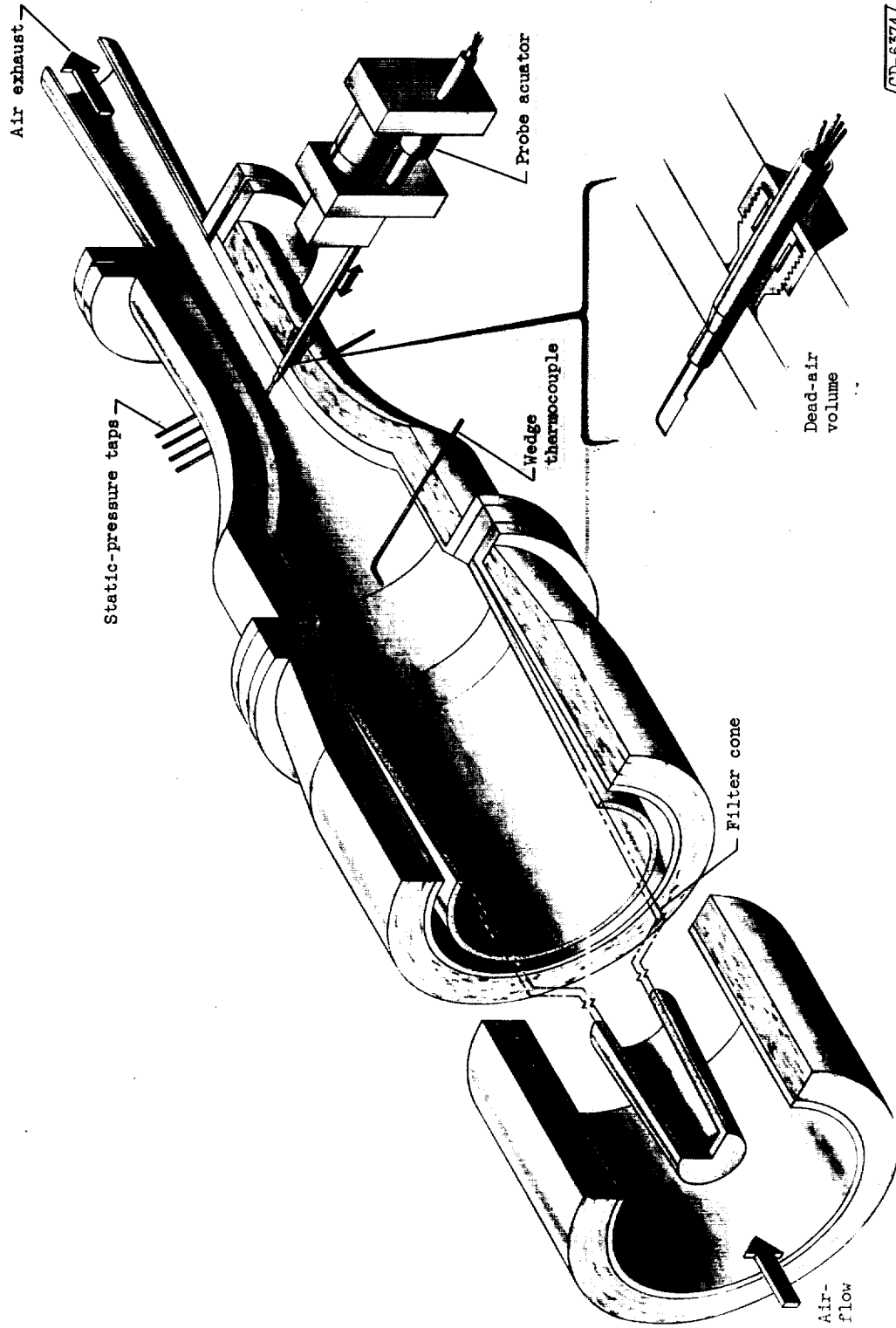


Figure 2. - Variable-density subsonic tunnel.

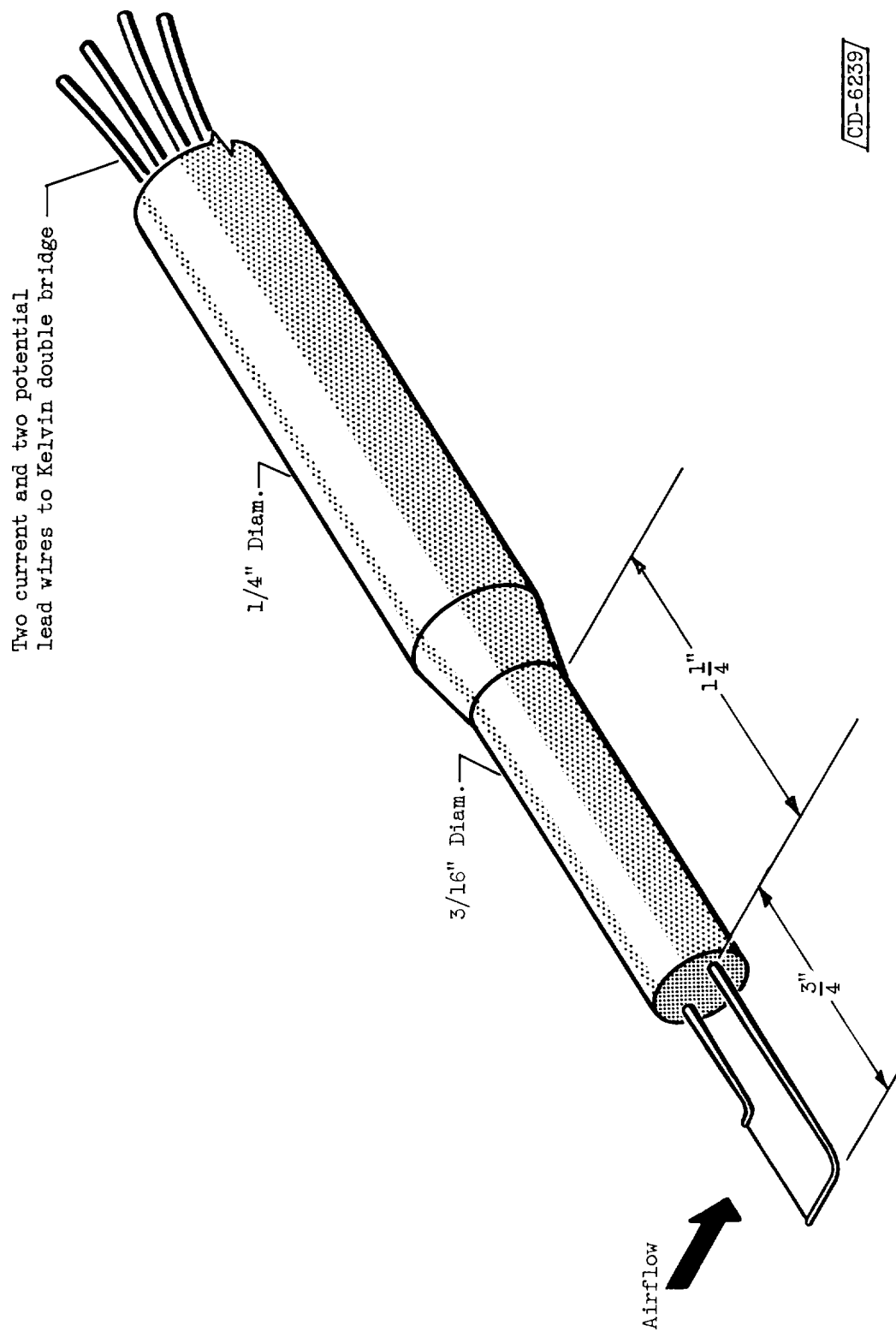
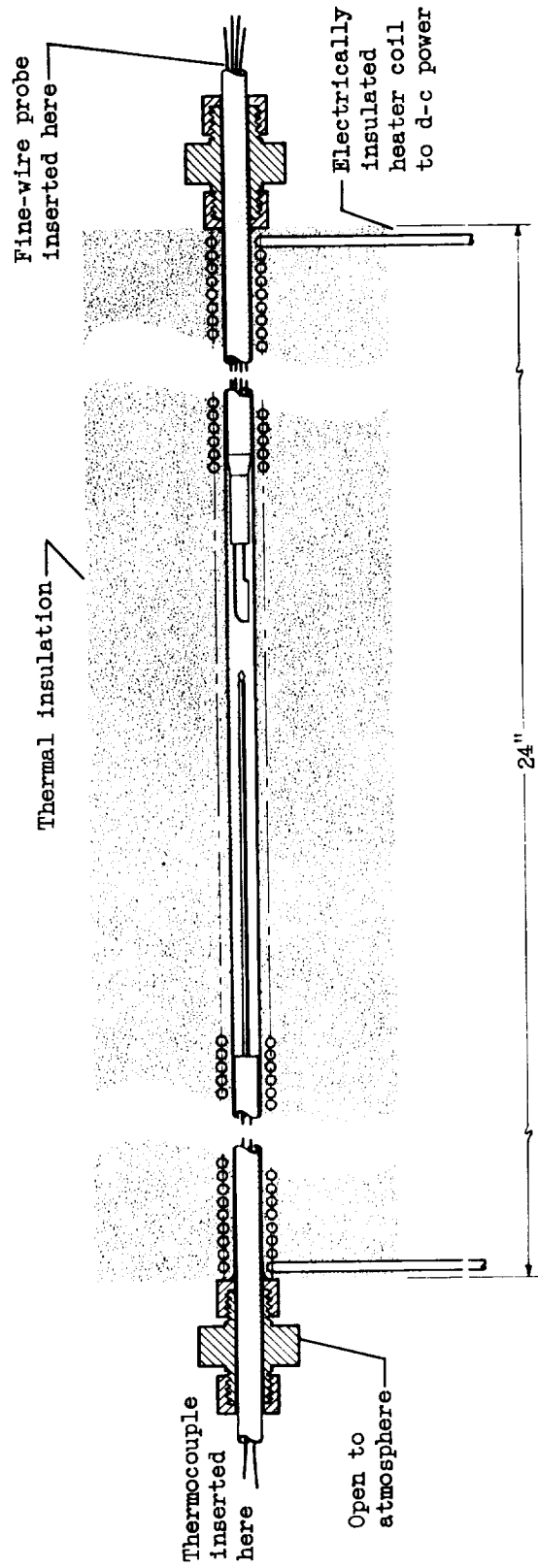


Figure 3. - Tungsten-wire supporting probe.



CD-6240

Figure 4. - Resistance-temperature calibration tank.

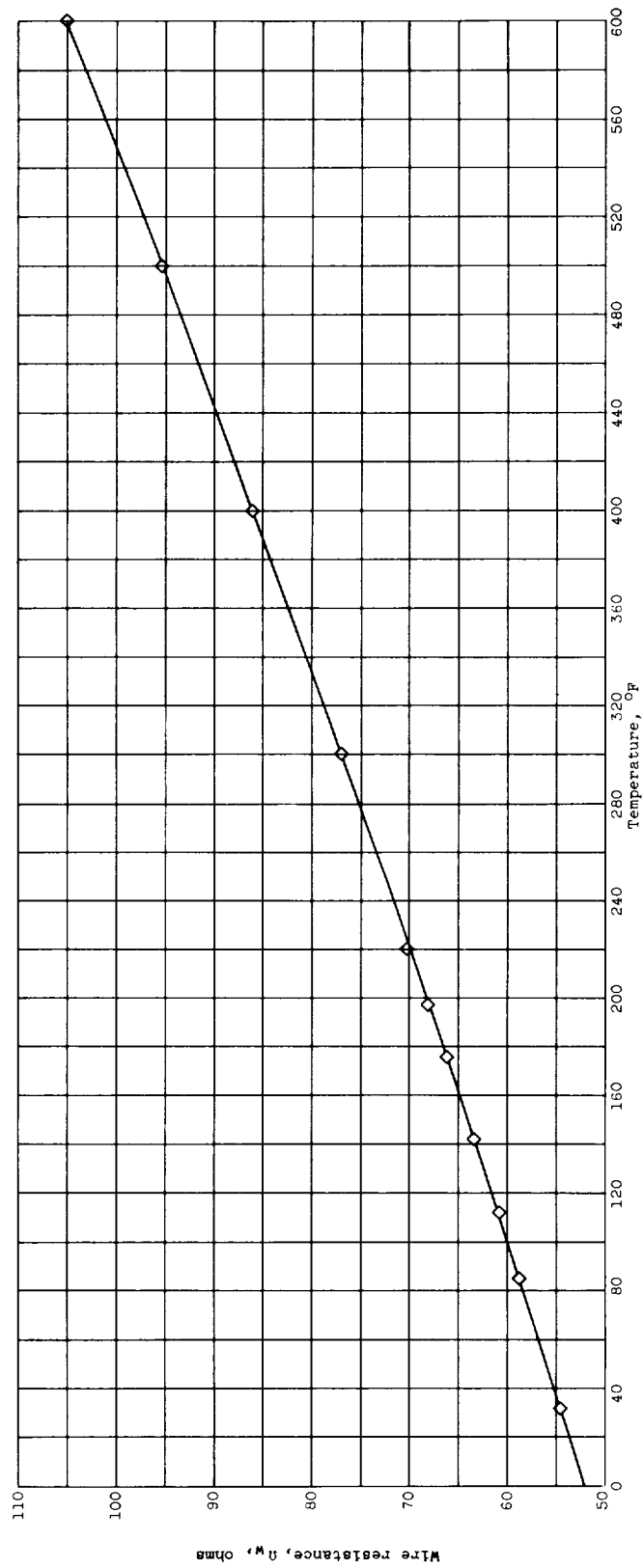


Figure 5. - Calibration of wire electrical resistance and temperature of probe 64; $\alpha_0 = 54.5$ ohms; $\alpha_w = 1.48 \times 10^{-3} \text{ } ^{\circ}\text{F}^{-1}$; $\beta_w = 2.67 \times 10^{-7} \text{ } ^{\circ}\text{F}^{-2}$.

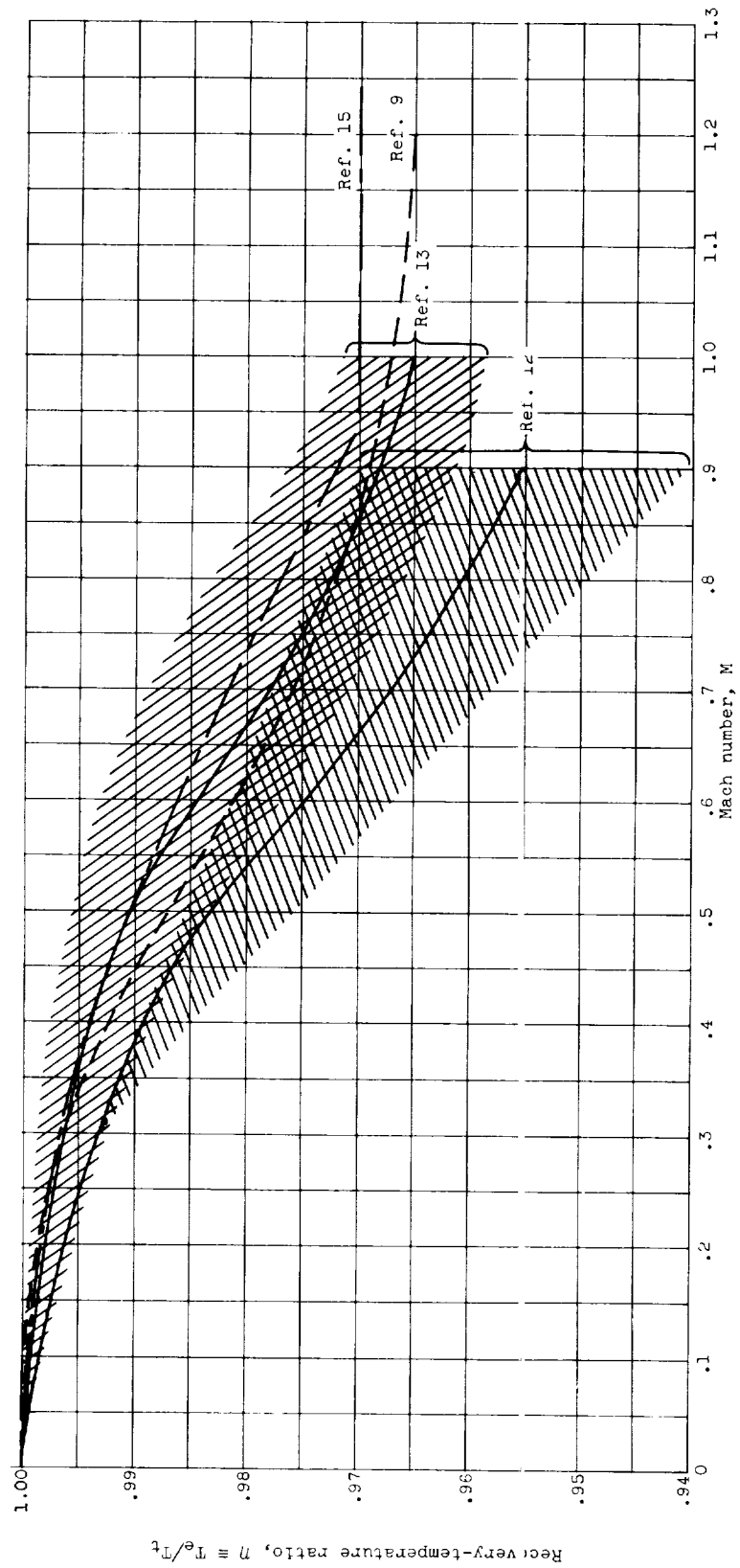


Figure 6. - Previous subsonic data on recovery temperature for Knudsen number less than 0.1.

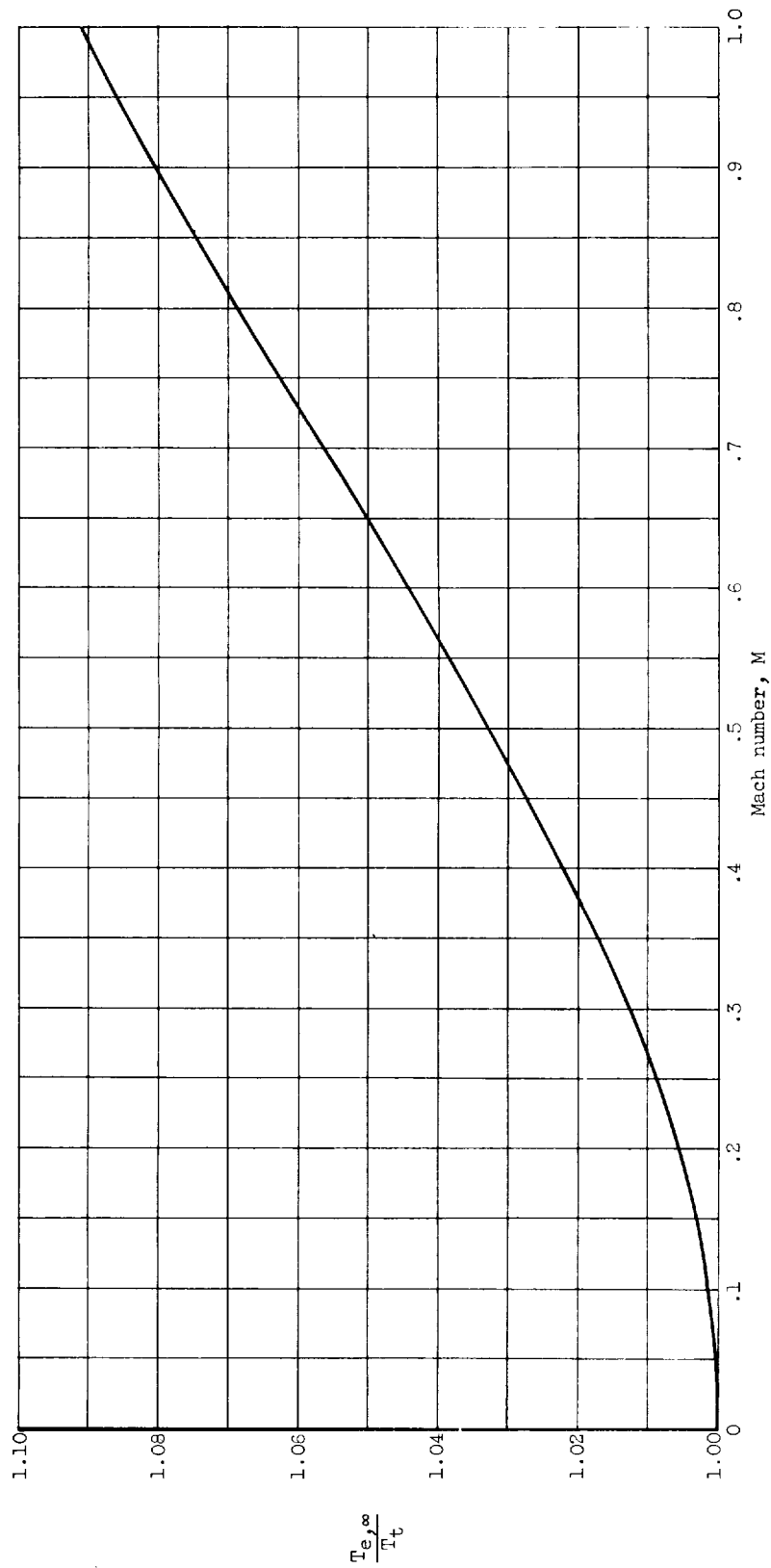


Figure 7. - Predicted curve from free-molecule-flow theory (eq. (8)).

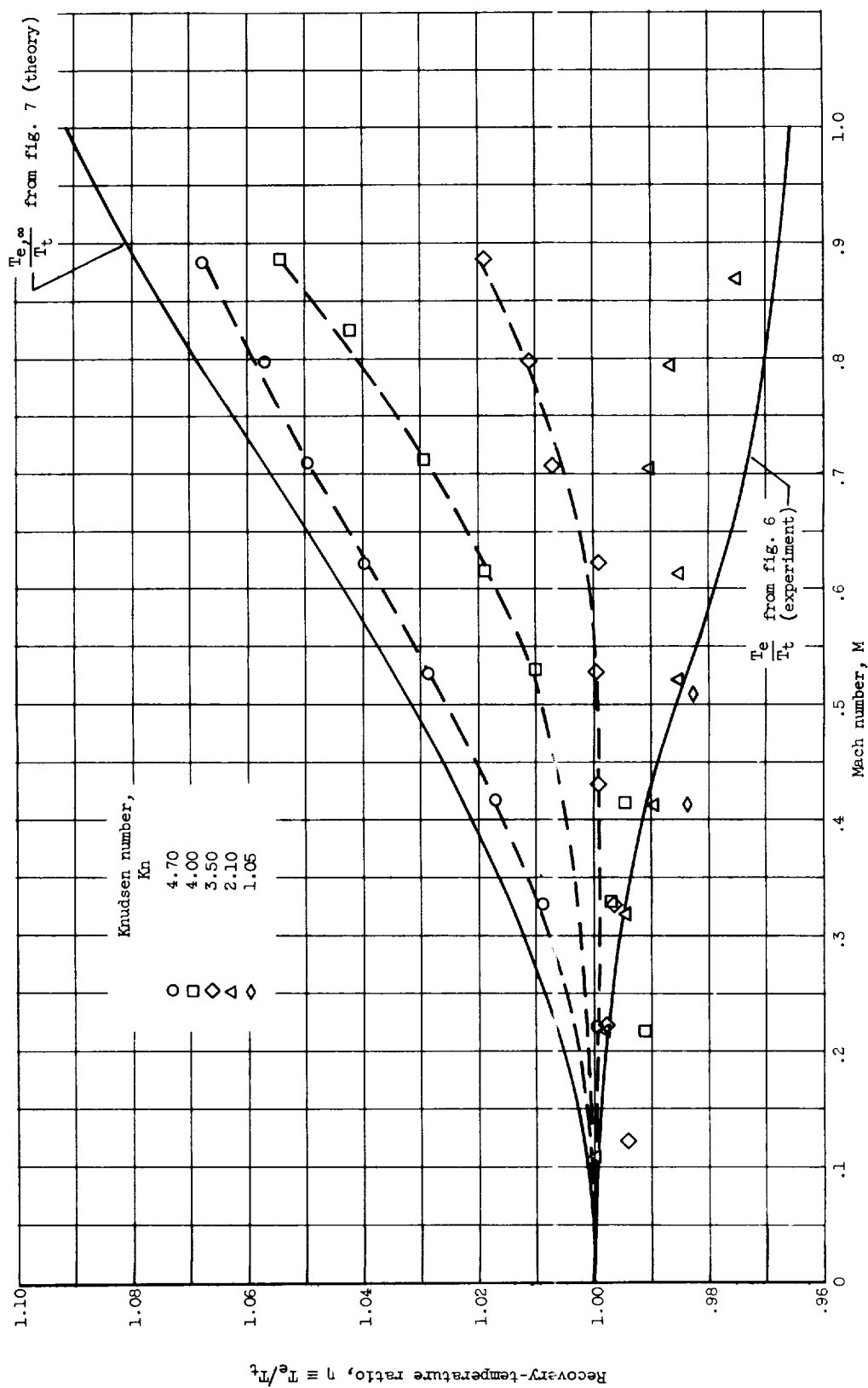


Figure 8. - Data of this research compared with previous data and theory.

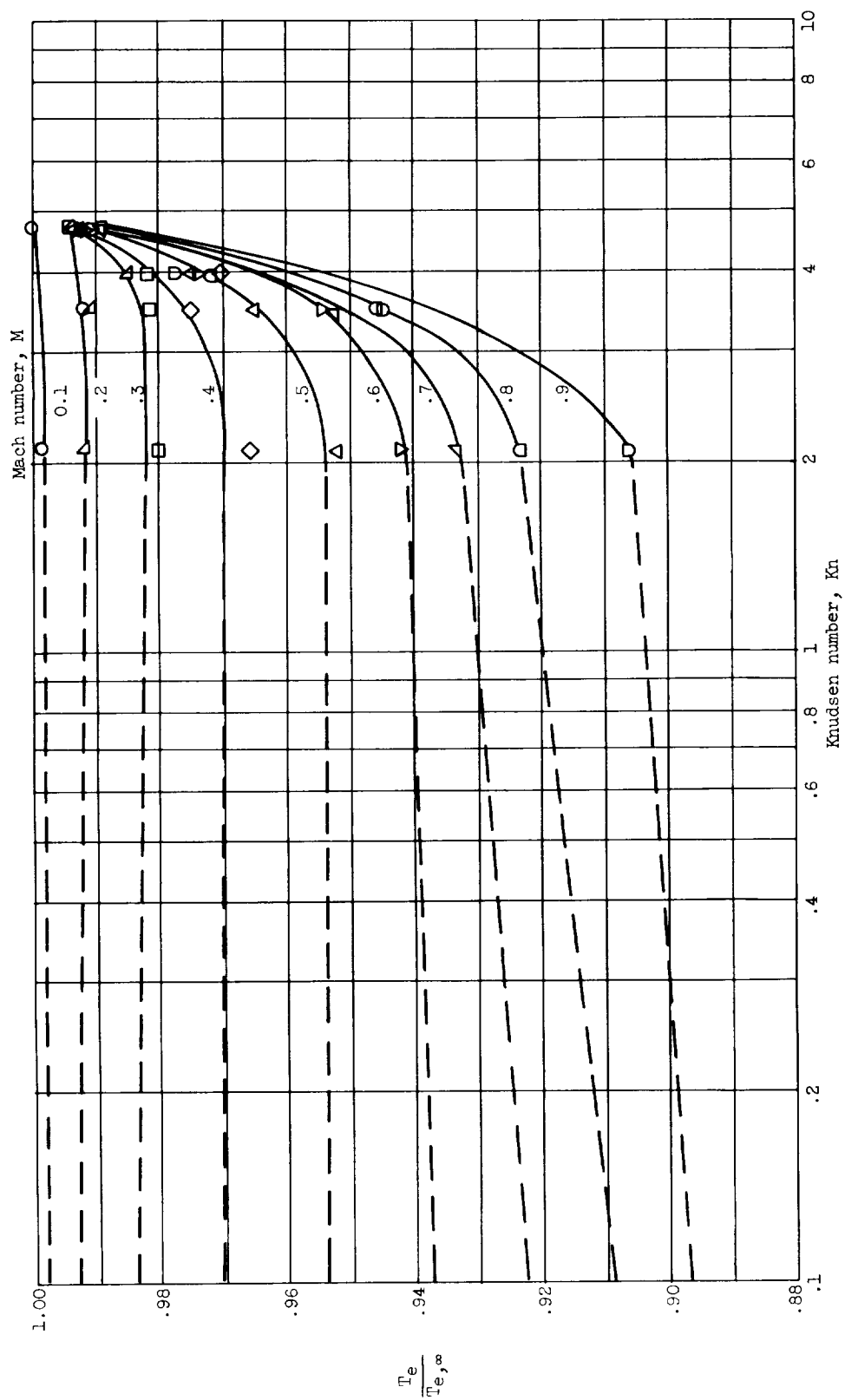


Figure 9. - Variation of ratio of measured to theoretical equilibrium temperature with free-stream Knudsen number for transverse cylinders. (Values of $T_e/T_{e,\infty}$ at $Kn = 0.1$ are from fig. 6 (ref. 9).)

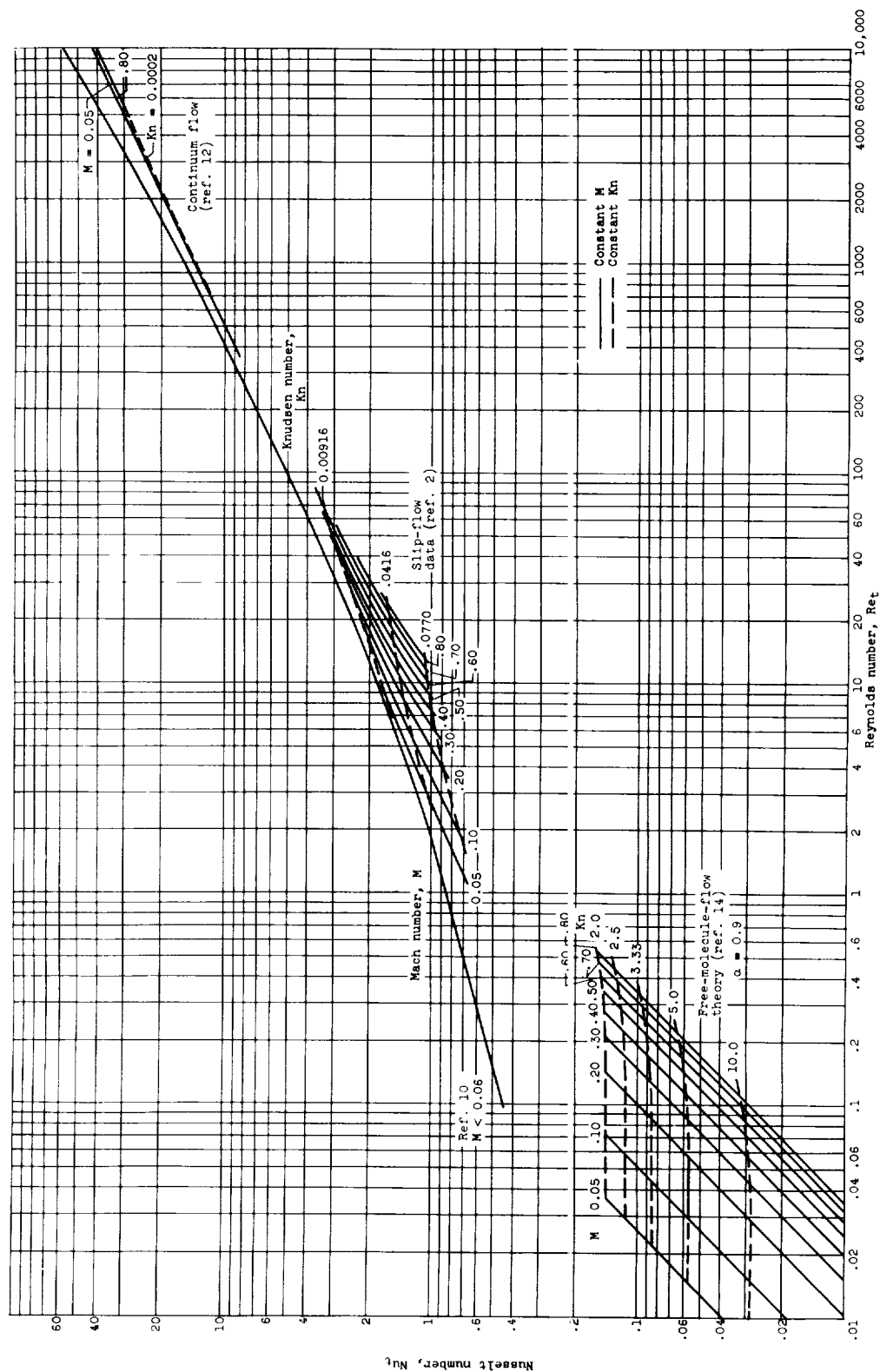


Figure 10. - Attempted Nusselt number correlation for cylinders in subsonic continuum, slip, and free-molecule flows.

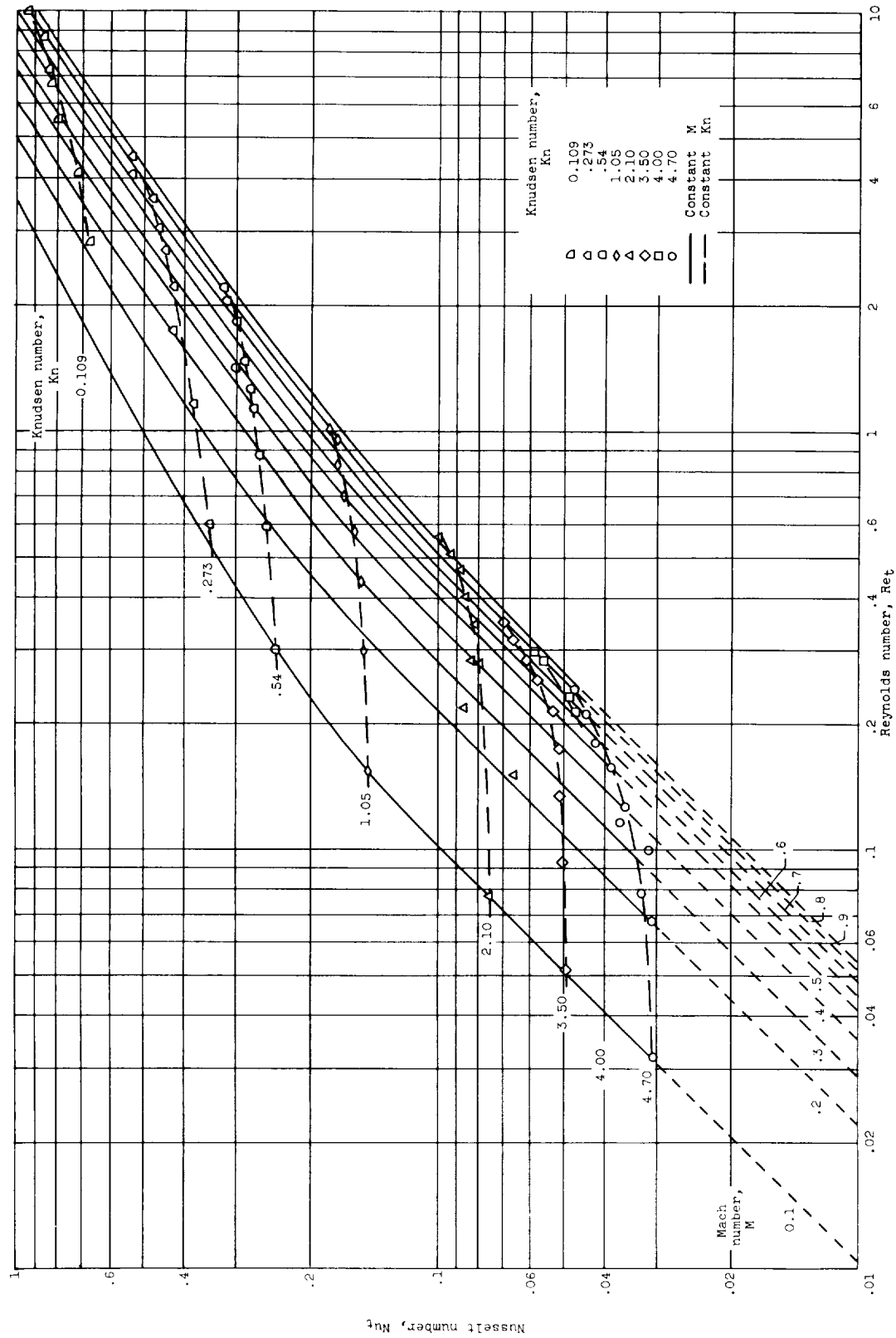
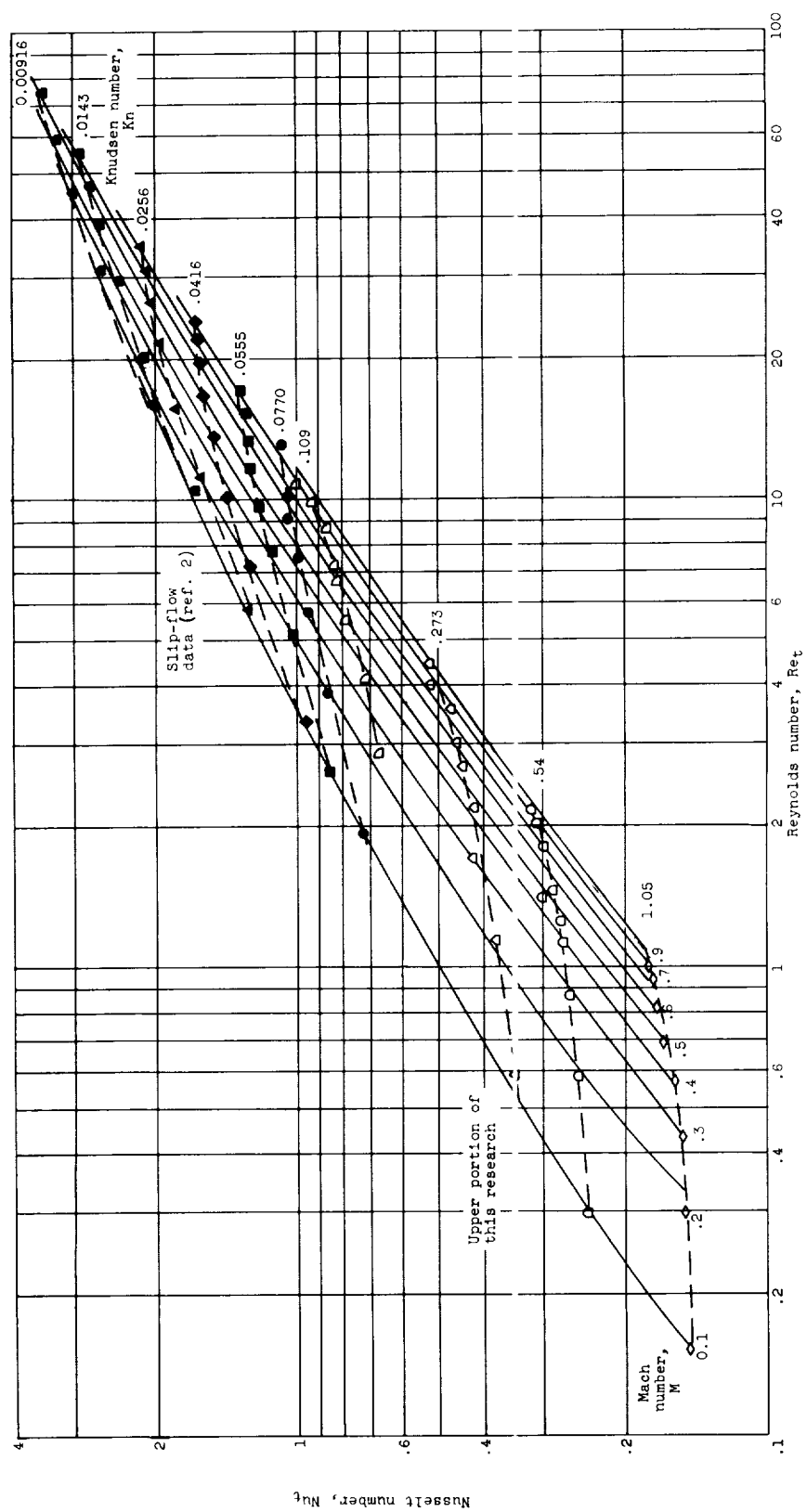


Figure 11. - Nusselt number correlation for cylinders in subsonic flow. Total air temperature, 80° F; length-average wire temperature, 580° F.



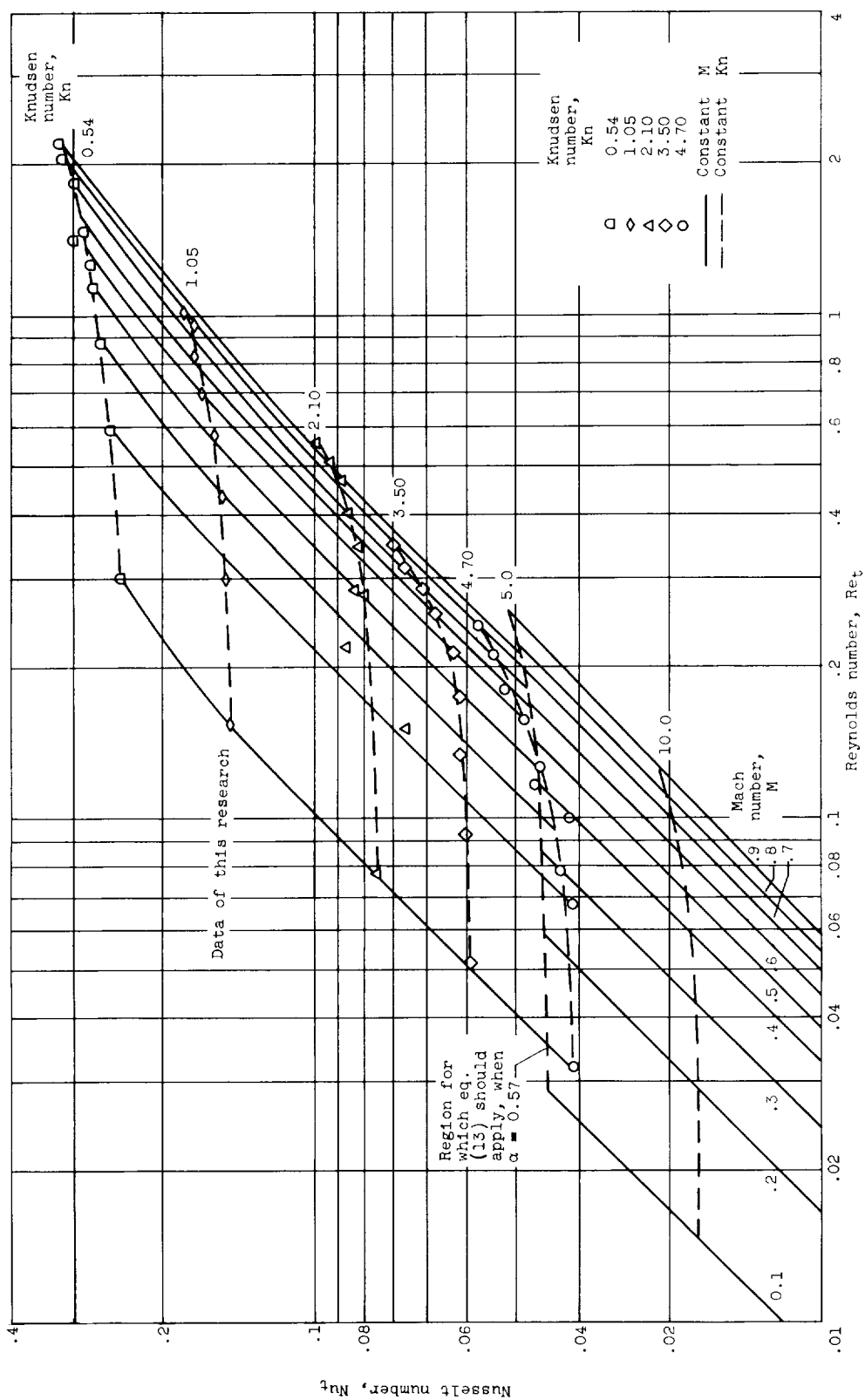


Figure 13. - Comparison of heat-transfer data with free-molecule-flow theory using $\alpha = 0.57$.

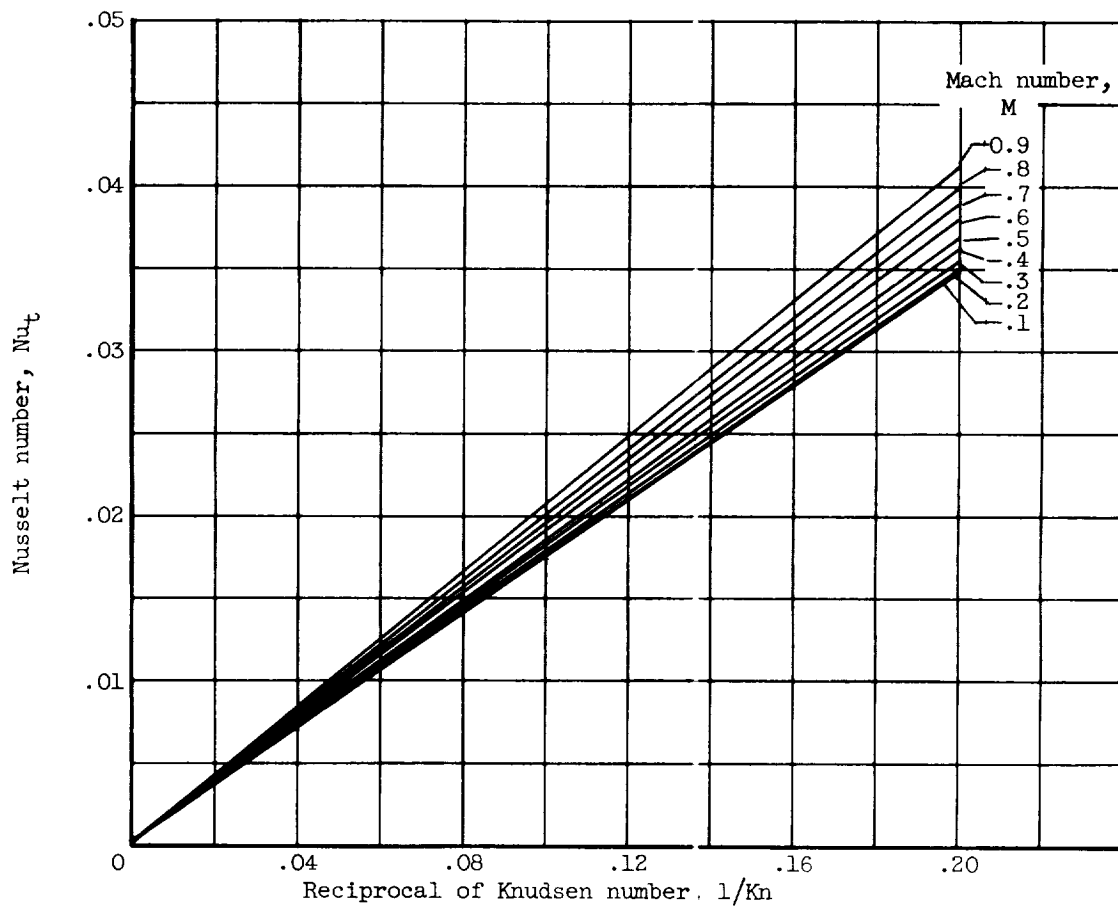


Figure 14. - Plot of equation (13) with $\alpha = 0.57$.

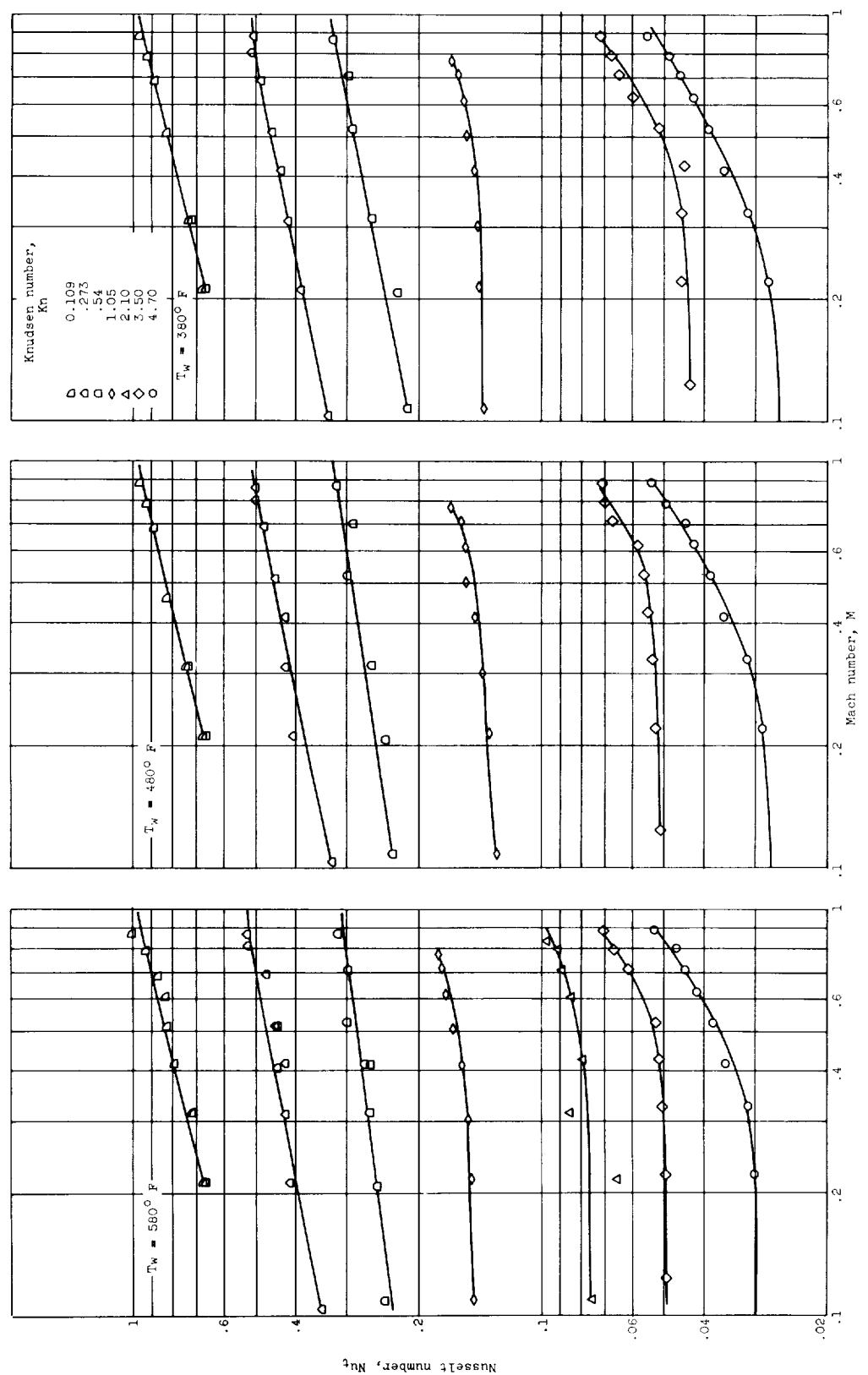


Figure 15. - Variation of Nusselt number with Mach number for constant Knudsen number and constant total air and wire temperatures. Total air temperature, 80° F.

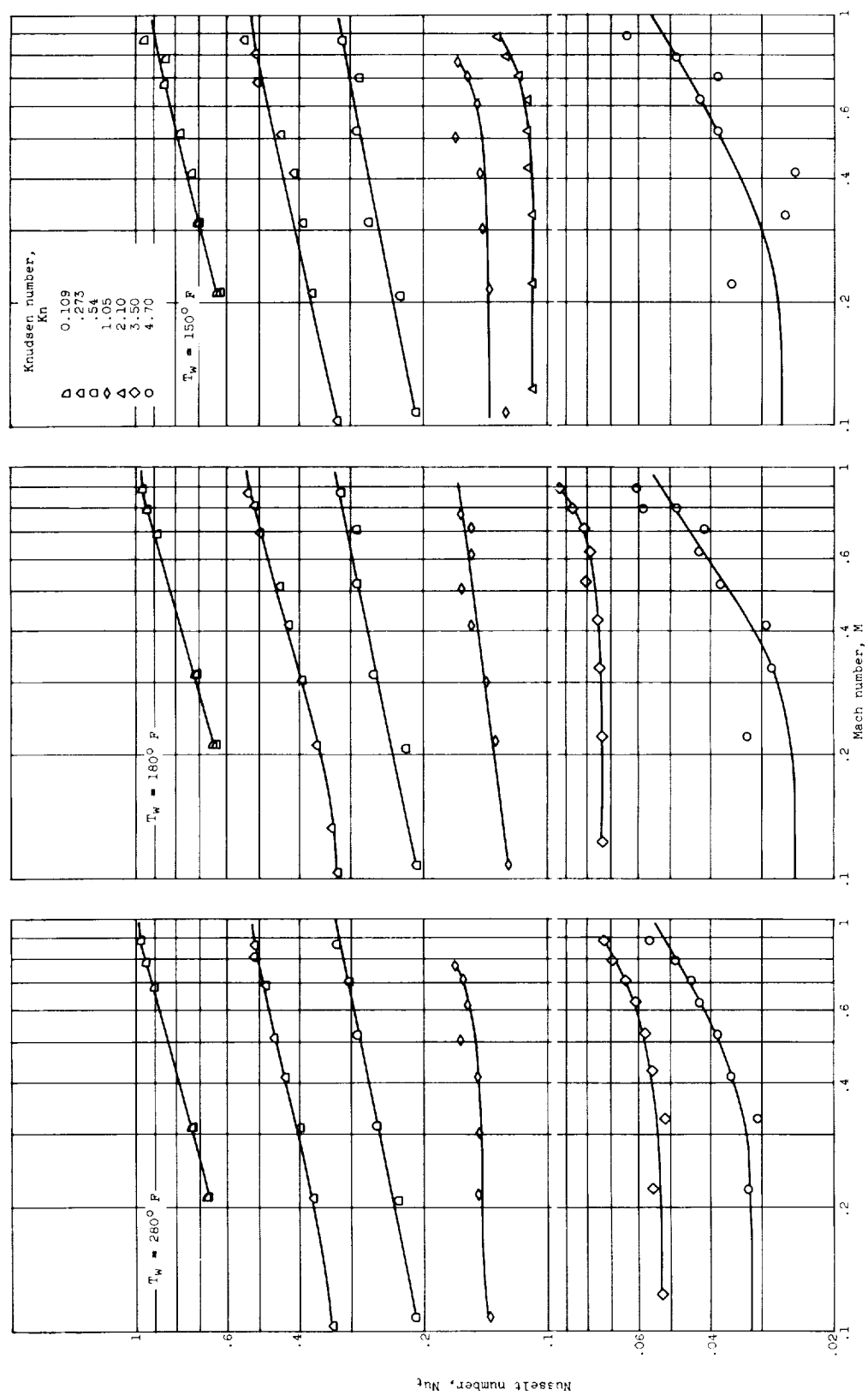


Figure 15. - Concluded. Variation of Nusselt number with Mach number for constant Knudsen number and constant total air and wire temperatures. Total air temperature, 80° F .

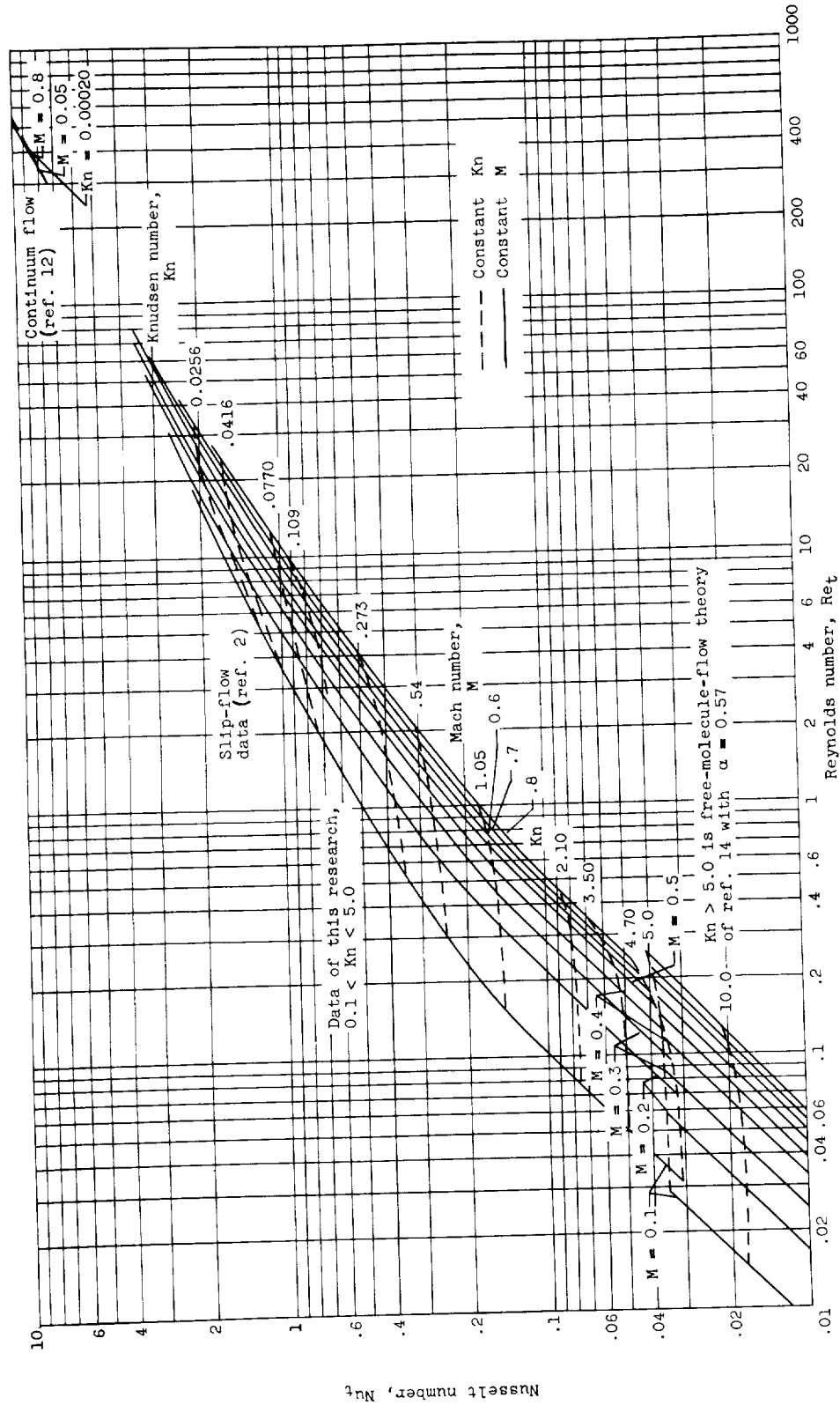


Figure 16. - Attempted Nusselt number correlation for cylinders in subsonic continuum, slip, and free-molecule flow using the data of this research and free-molecule-flow theory with $\alpha = 0.57$.

